Flux vacua in String Theory, generalized calibrations and supersymmetry breaking

Luca Martucci

Workshop on complex geometry and supersymmetry, IPMU
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Plan of this talk

- Generalized calibrations and fluxes
- Generalized calibrations and SUSY vacua
- Calibrations, SUSY-breaking and 4D potential

Generalized calibrations and fluxes

Ordinary calibrations

Harvey & Lawson `82

- An **ordinary calibration is a** differential p-form ω such that

  1) Algebraic condition: $\omega|_\Sigma \leq \sqrt{g|_\Sigma}\,\mathrm{d}\sigma$ for any p-dim submanifold Σ

  2) Differential condition: $\mathrm{d}\omega = 0$

- **Calibrated** submanifolds: $\omega|_\Sigma = \sqrt{g|_\Sigma}\,\mathrm{d}\sigma$

Ordinary calibrations

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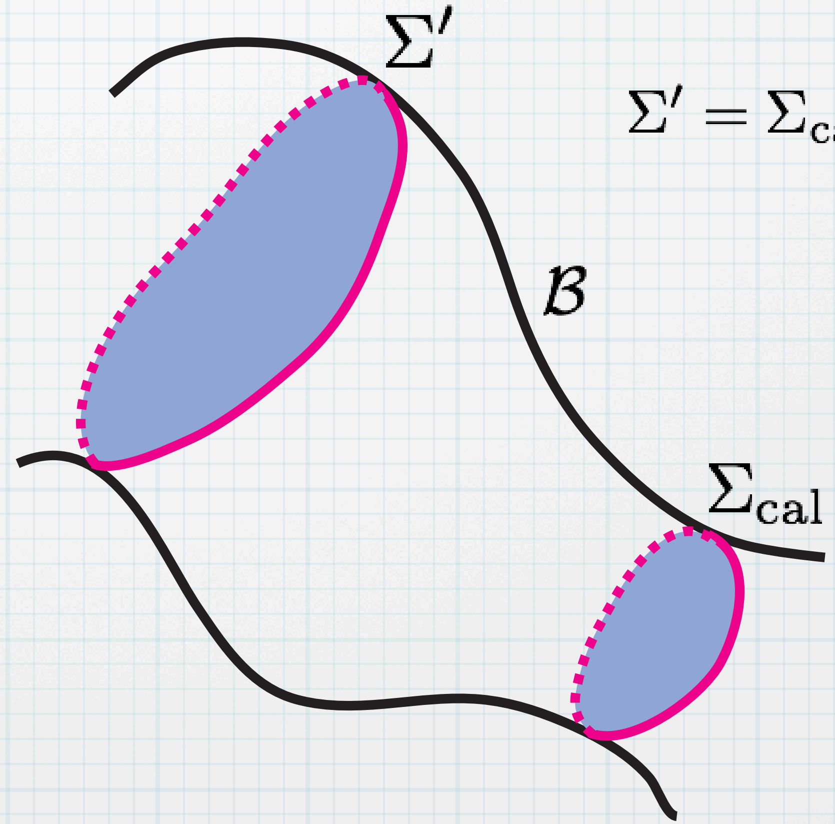
- **Main property:** calibrated cycles are volume minimizing

$$\mathrm{Vol}(\Sigma') \geq \mathrm{Vol}(\Sigma_{\mathrm{cal}})$$



$$\Sigma' = \Sigma_{\mathrm{cal}} + \partial\mathcal{B}$$

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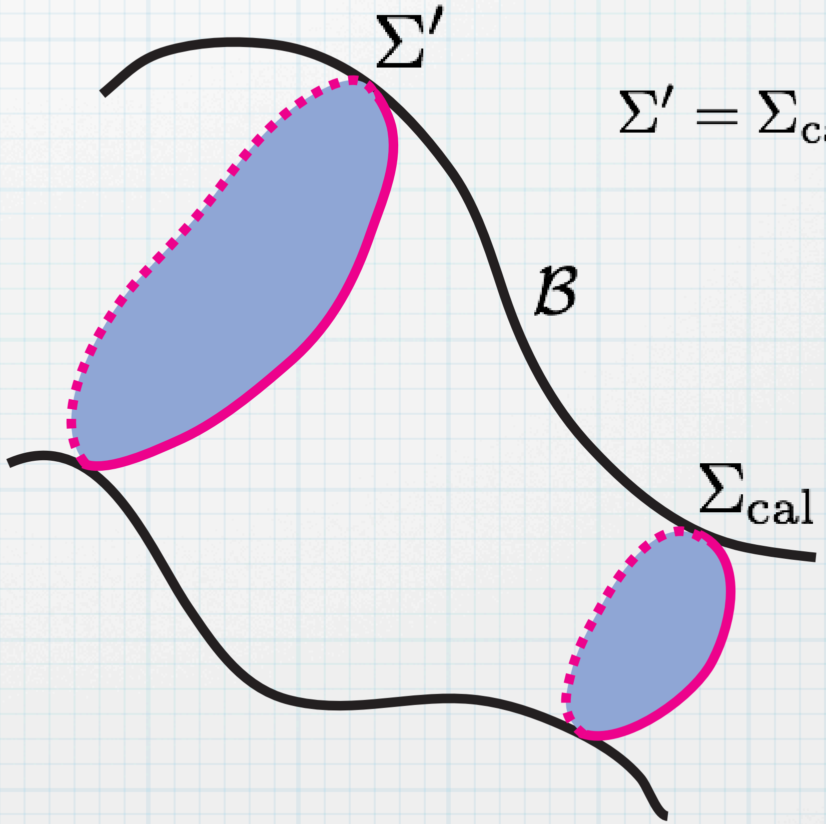

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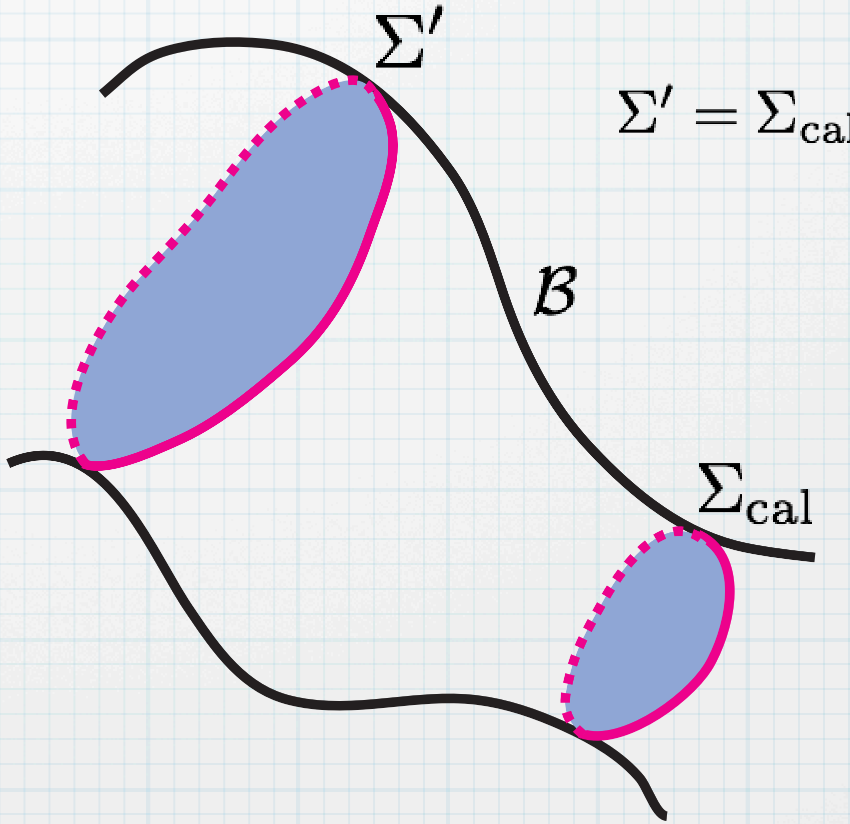
algebraic condition → $$\geq \int_{\Sigma'} \omega$$



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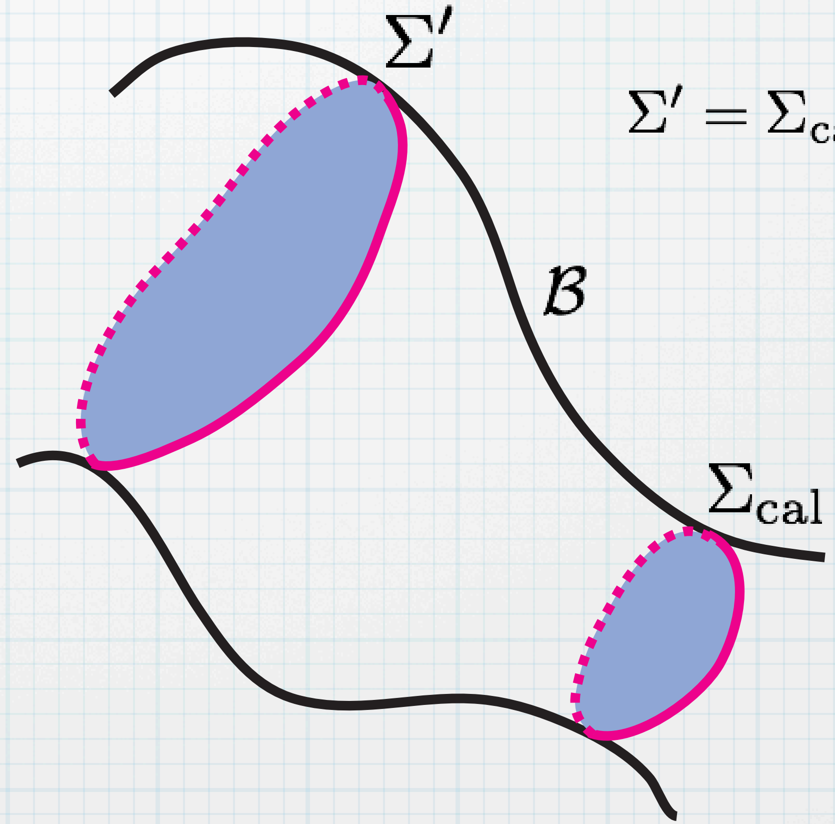

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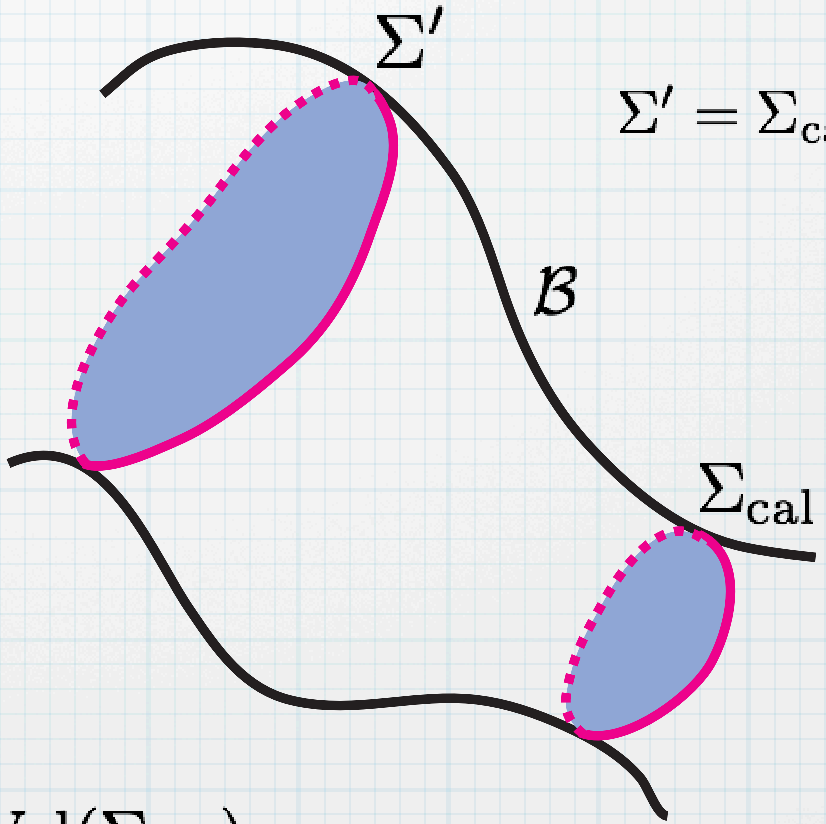
calibration condition → $$= \int_{\Sigma_{\rm cal}} \sqrt{g|_{\Sigma_{\rm cal}}}\, \mathrm{d}\sigma = \mathrm{Vol}(\Sigma_{\rm cal})$$



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- Explicitly realized by calibrations

  * bulk SUSY $\longrightarrow$ $\exists$ p-calibration ω
  * brane SUSY $\longrightarrow$ $\Sigma_{\rm SUSY}\equiv\Sigma_{\rm cal}$

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IIA IIB

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$$\mathcal{E}_{\mathrm{DBI}}(\Sigma, \mathcal{F})$$

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Generalized calibrations

Koerber `05; L.M. & Smyth `05

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- A generalized calibration is a background polyform

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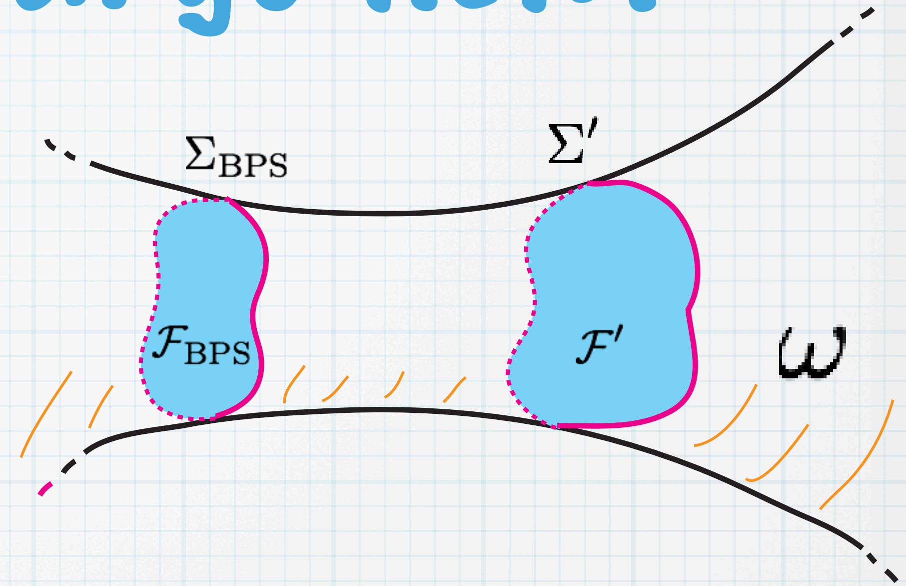
- **Calibrated** (or **BPS**) D-branes: $\mathcal{E}_{\rm DBI}(\Sigma,\mathcal{F}) = \left[\omega|_\Sigma \wedge e^{\mathcal{F}}\right]_{\rm top}$

The stability argument

- Main property: **calibrated D-branes are stable!**

$$E(\Sigma', \mathcal{F}') \geq E(\Sigma, \mathcal{F})_{\mathrm{BPS}}$$



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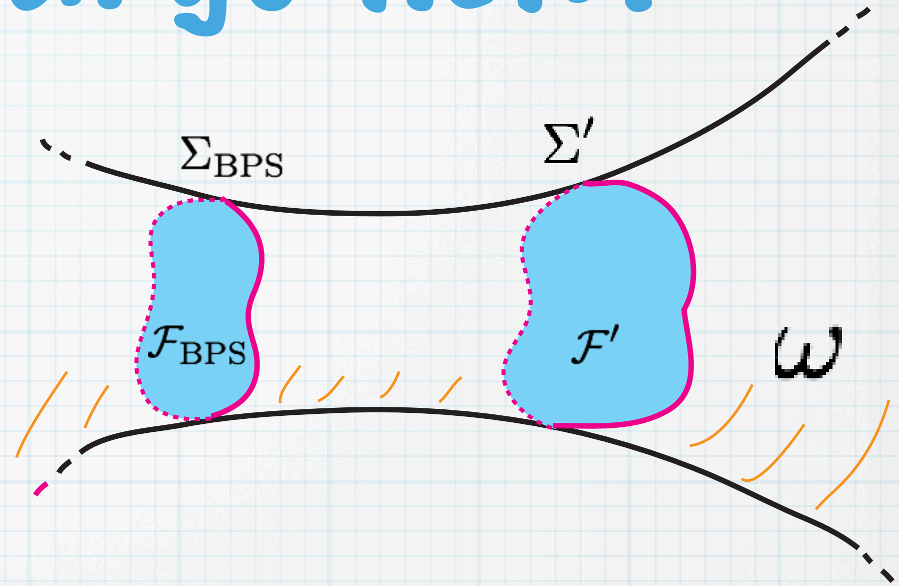

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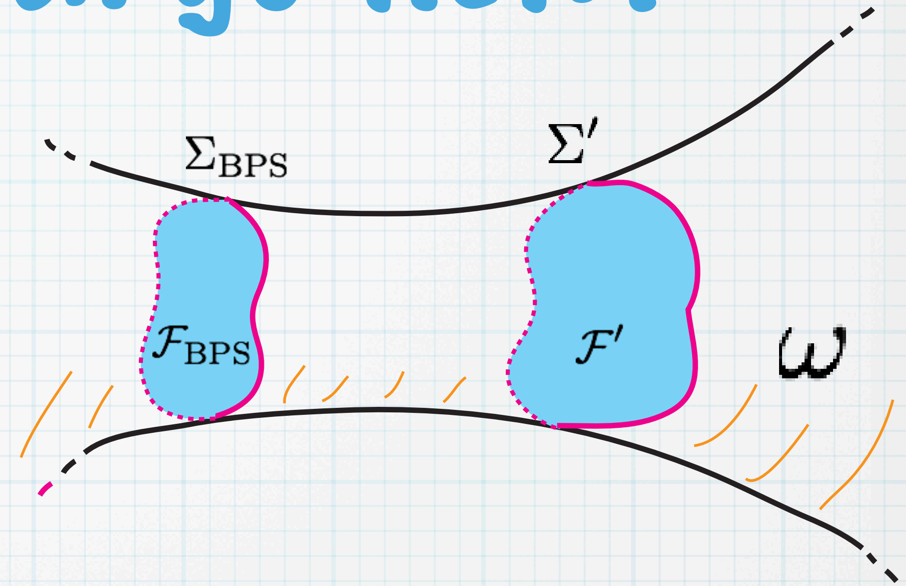

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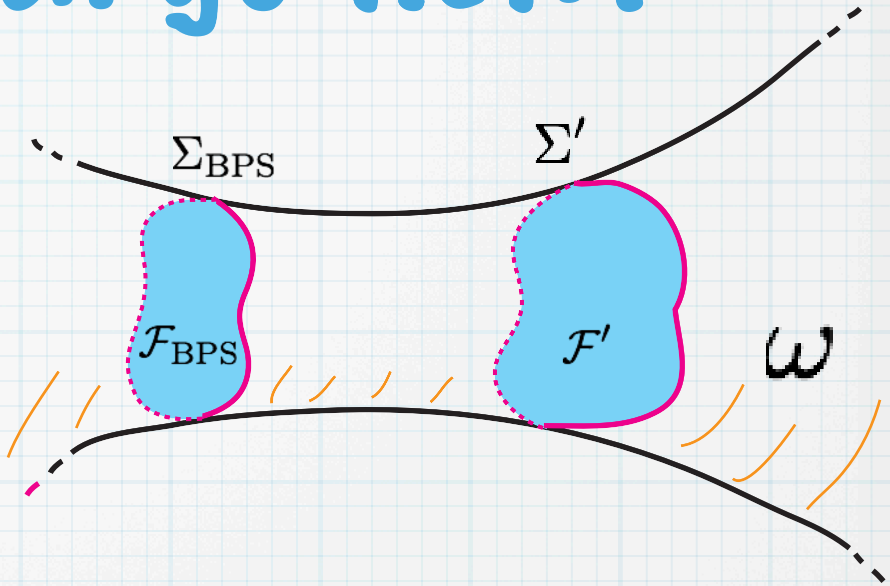

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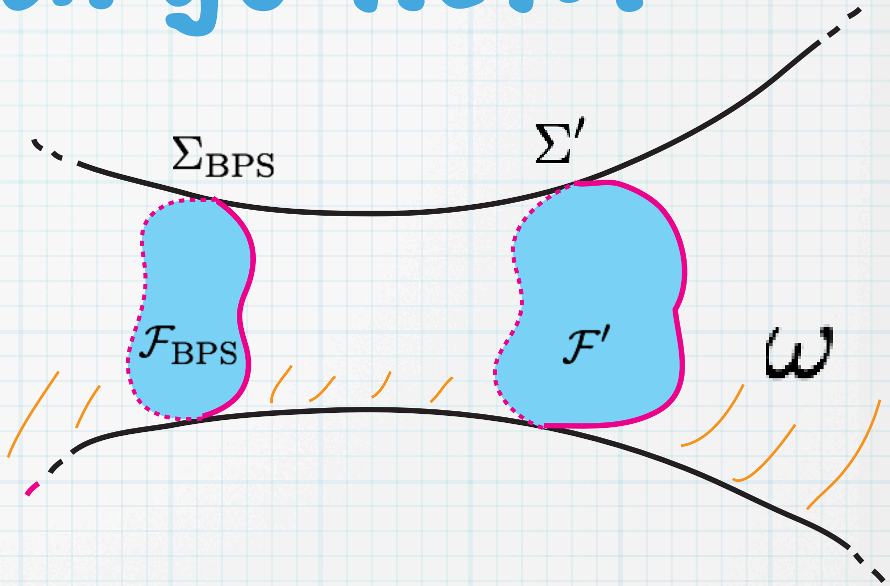

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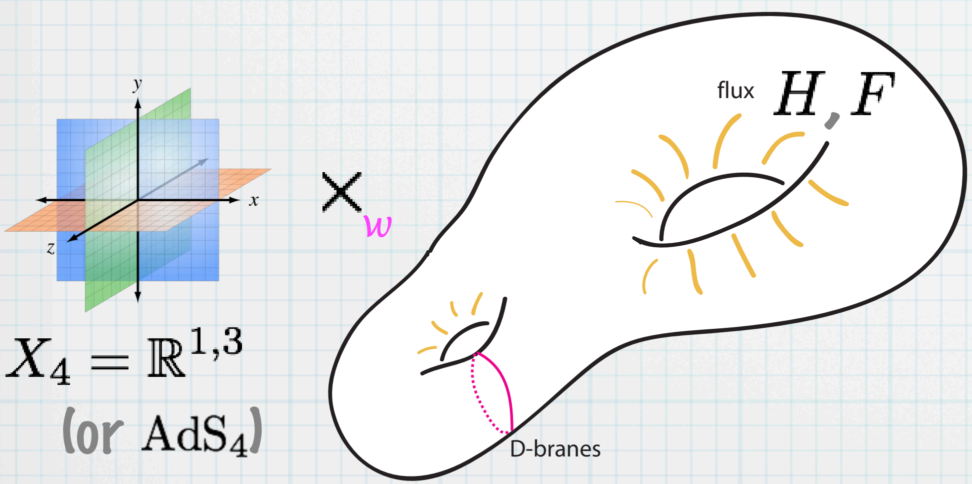
D-brane calibration condition → $$= \int_{\Sigma_{\mathrm{BPS}}} [\mathcal{E}_{\mathrm{DBI}}(\Sigma, \mathcal{F})_{\mathrm{BPS}} + \mathcal{E}_{\mathrm{WZ}}(\Sigma, \mathcal{F})_{\mathrm{BPS}}] = E(\Sigma, \mathcal{F})_{\mathrm{BPS}}$$

Generalized calibrations and type II SUSY vacua

N=1 vacua and pure spinors

- General 4+6 backgrounds:

$$ds^2 = e^{2A(y)} g_4(x) + g_6(y) \quad \text{plus} \quad H(y)\ ,\quad \Phi(y)\ ,\quad F(y)$$



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- Ordinary Killing spinors:

$$\epsilon_1 = \zeta \otimes \eta_1 + \text{ c.c.}$$

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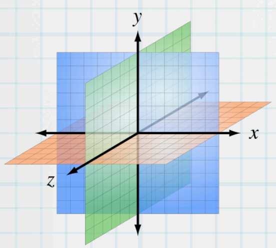
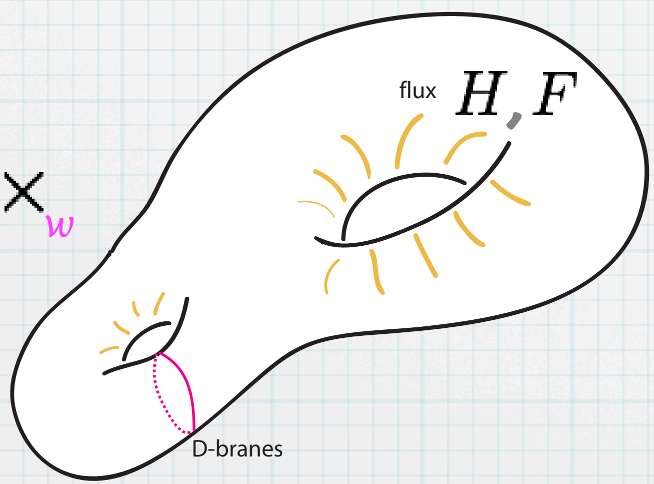


$X_4 = \mathbb{R}^{1,3}$

(or AdS_4)

$\times_w$



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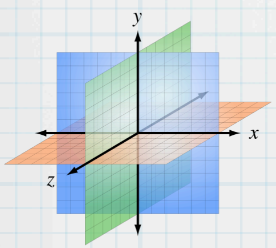
SO(6,6) pure spinors

$$t \simeq e^{-\Phi} \eta_1 \otimes \eta_2^\dagger$$

$$\mathcal{Z} \simeq e^{3A-\Phi} \eta_1 \otimes \eta_2^T$$



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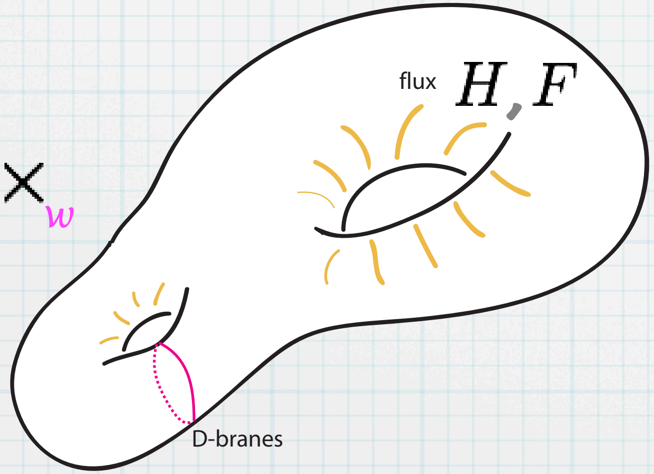

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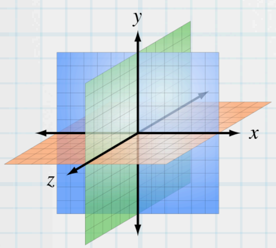

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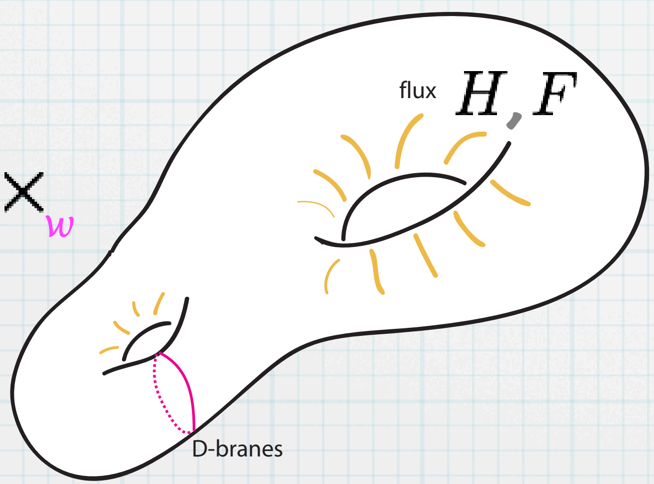

in IIA

$$t = t_1 + t_3 + t_5$$

$$\mathcal{Z} = \mathcal{Z}_0 + \mathcal{Z}_2 + \mathcal{Z}_4 + \mathcal{Z}_6$$

in IIB

$$t = t_0 + t_2 + t_4 + t_6$$

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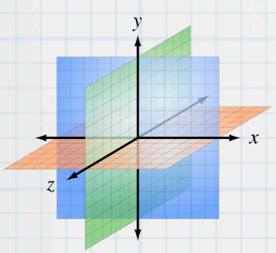

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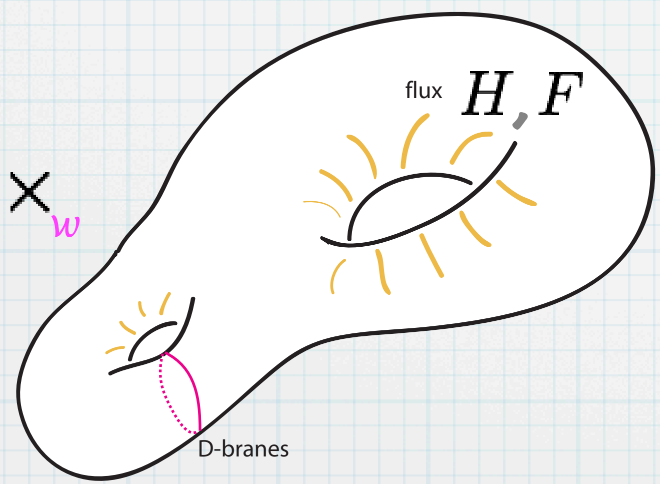

- t and $\mathcal{Z}$ contain complete information about:

  * NS sector: g_6 , Φ and A

    (H excluded)

  * Internal spinors: η_1 and η_2

N=1 vacua and calibrations

L.M. & Smyth `05

SUSY conditions

Graña, Minasian, Petrini & Tomasiello `05

* $\mathrm{d}_H(e^{4A}\mathrm{Re}\, t) = e^{4A} * F$
* $\mathrm{d}_H(e^{2A}\mathrm{Im} t) = 0$
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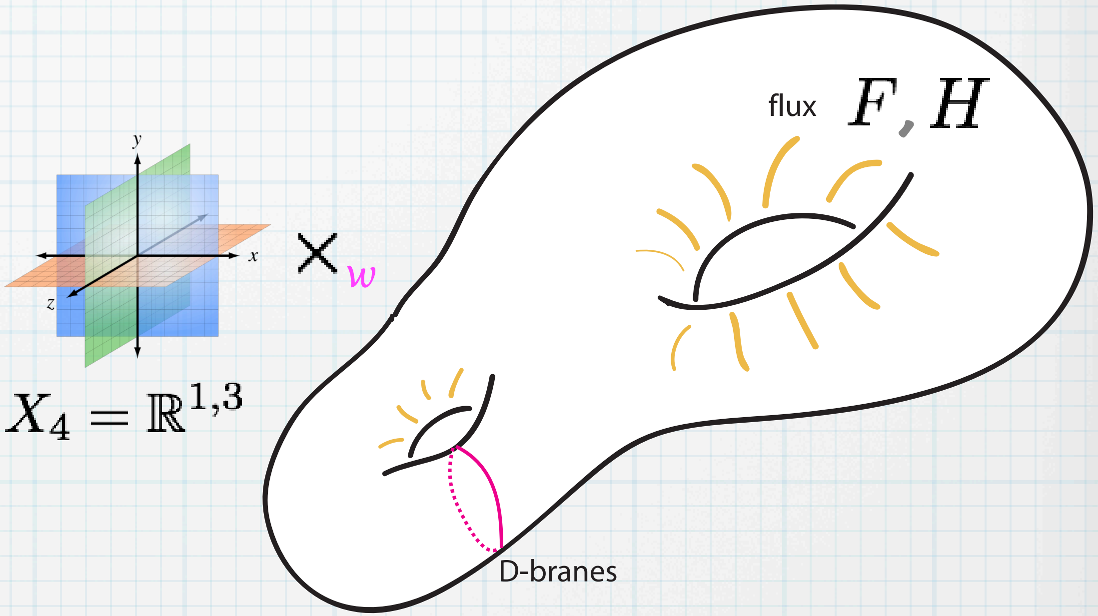

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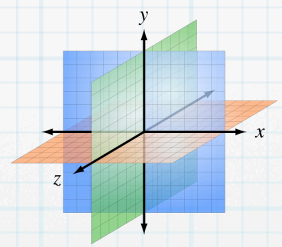
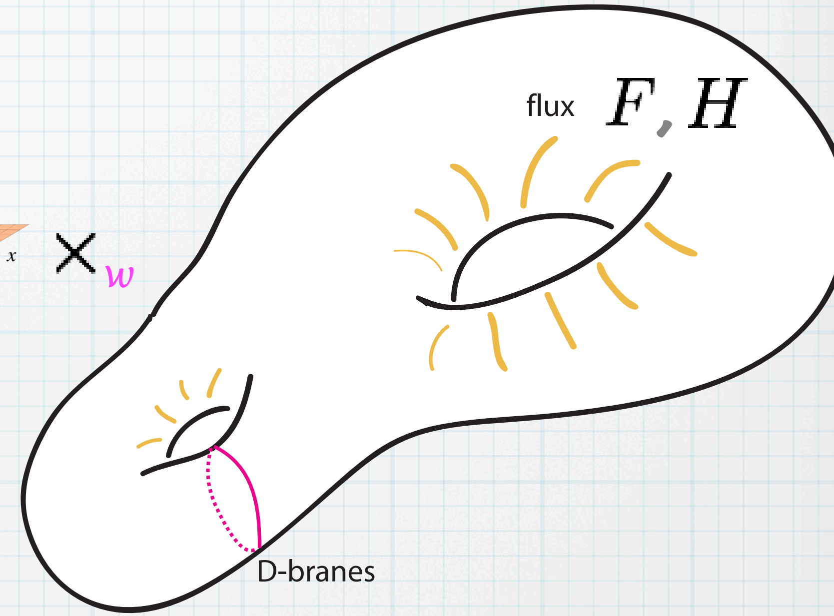


$X_4 = \mathbb{R}^{1,3}$ $\times_w$



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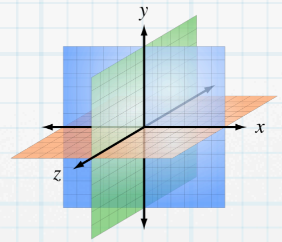

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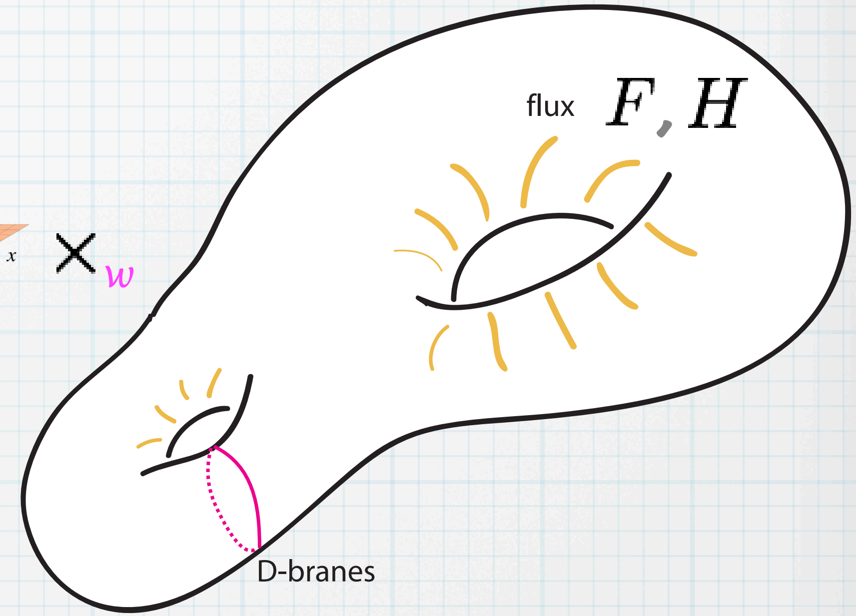

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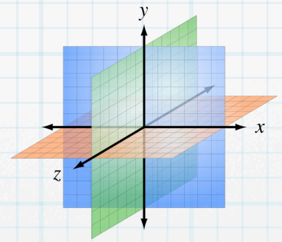

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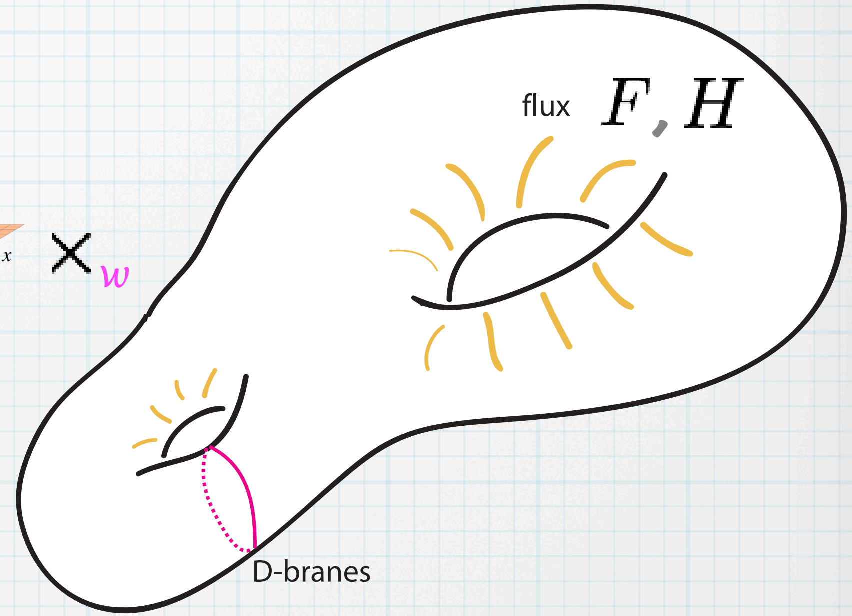
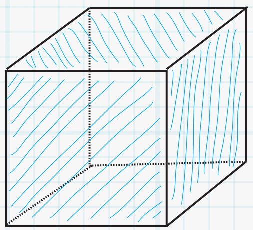


They have a natural interpretation in terms of D-brane **generalized calibrations!**

N=1 vacua and calibrations

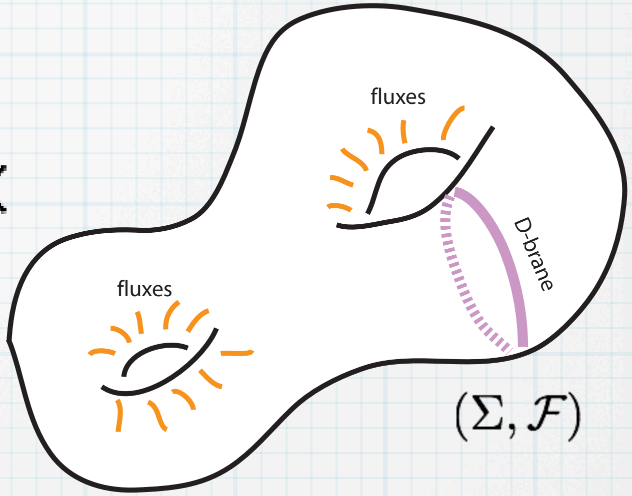
L.M. & Smyth `05

- $e^{4A}\mathrm{Re}\, t$ is a **generalized calibration** for **space-filling** D-branes



space-filling

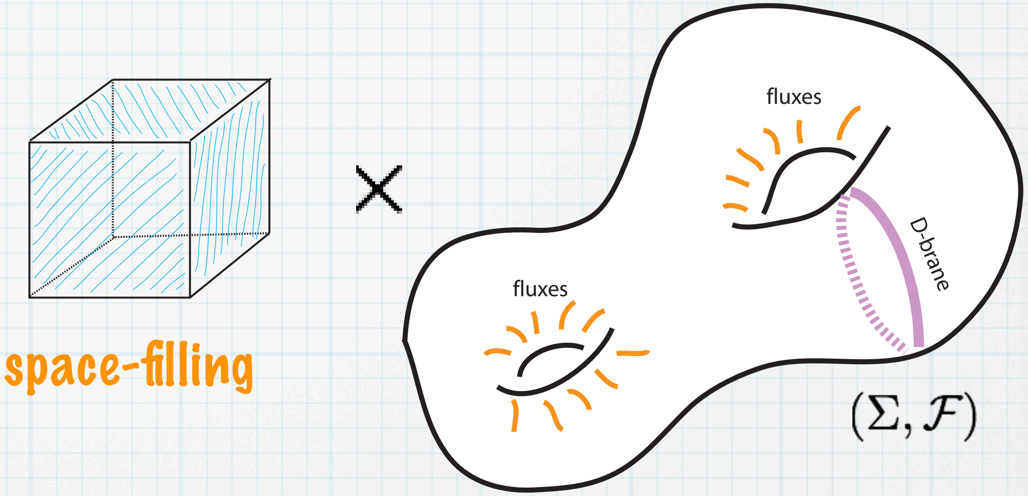
×



N=1 vacua and calibrations

L.M. & Smyth `05

- $e^{4A}\mathrm{Re}\, t$ is a **generalized calibration** for **space-filling** D-branes



- For SUSY (**BPS**) **space-filling** D-branes, internal $(\Sigma, \mathcal{F})$ satisfies:

$$e^{-\Phi}\sqrt{g|_\Sigma + \mathcal{F}}\,\mathrm{d}\sigma = [\mathrm{Re}\, t|_\Sigma \wedge e^{\mathcal{F}}]_{\mathrm{top}}$$

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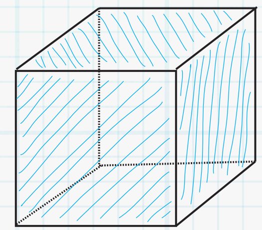
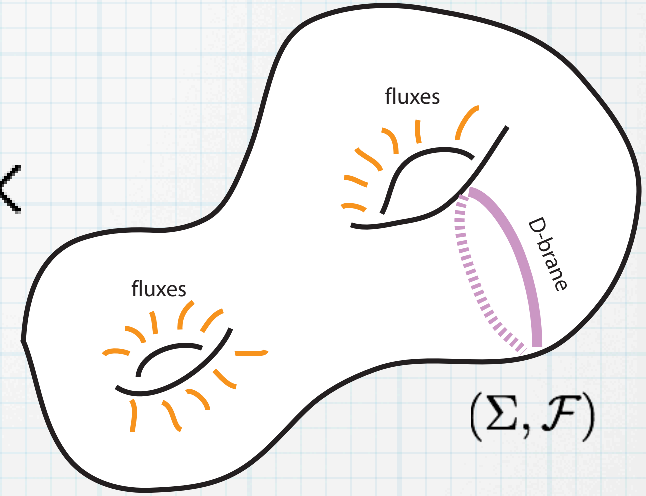


space-filling

$\times$



- For SUSY (**BPS**) **space-filling** D-branes, internal $(\Sigma, \mathcal{F})$ satisfies:

$$e^{-\Phi}\sqrt{g|_\Sigma + \mathcal{F}}\, d\sigma = [\mathrm{Re}\, t|_\Sigma \wedge e^{\mathcal{F}}]_{\mathrm{top}}$$

  - $(\Sigma, \mathcal{F})$ is **generalized complex submanifold** w.r.t. GCS $\mathcal{J}$ defined by $\mathcal{Z}$ **(F-flatness)**
  - $(\Sigma, \mathcal{F})$ satisfies the **`speciality'** condition $$[\mathrm{Im}\, t|_\Sigma \wedge e^{\mathcal{F}}]_{\mathrm{top}} = 0$$ **(D-flatness)**

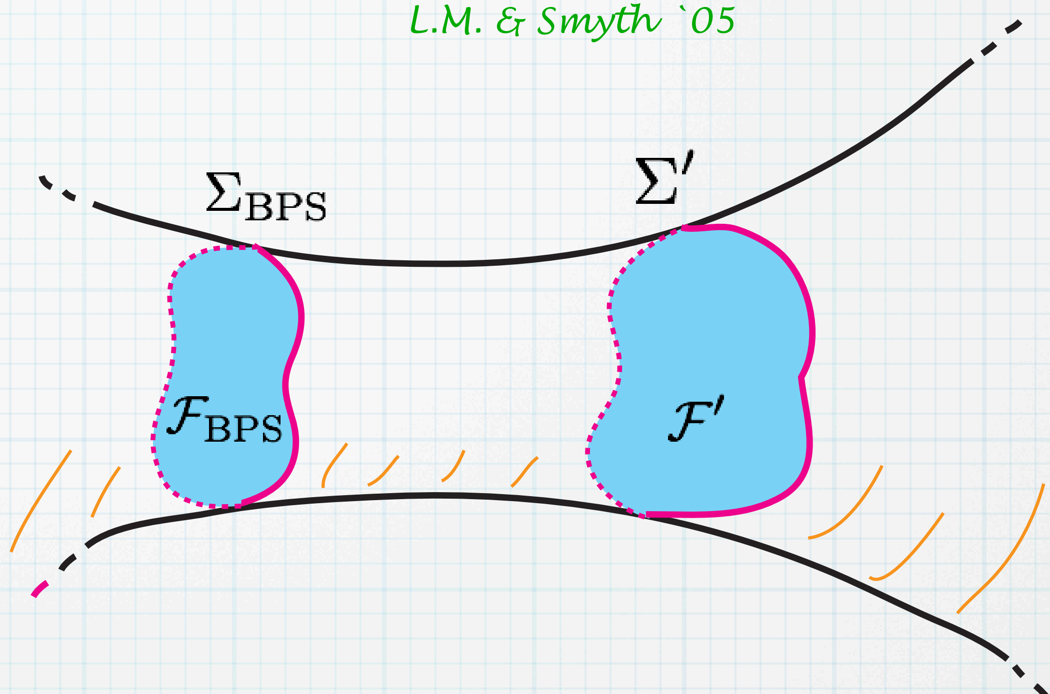
N=1 vacua and calibrations

L.M. & Smyth `05

- **BPS lower bound** for D-brane energy

$$V(\Sigma, \mathcal{F})_{\rm BPS} \leq V(\Sigma', \mathcal{F}')$$

no open-string tachyons!



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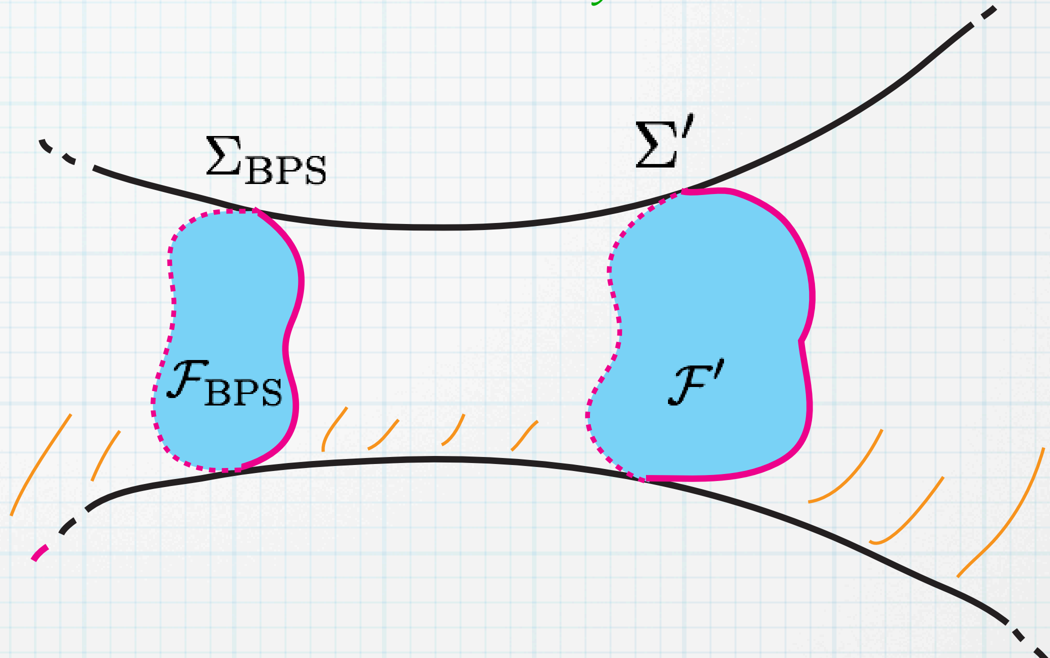


- For space-filling D-branes, it originates in bulk SUSY condition

$$\boxed{d_H(e^{4A}{\rm Re}\,t) = e^{4A} * F}$$

Gauge BPSness

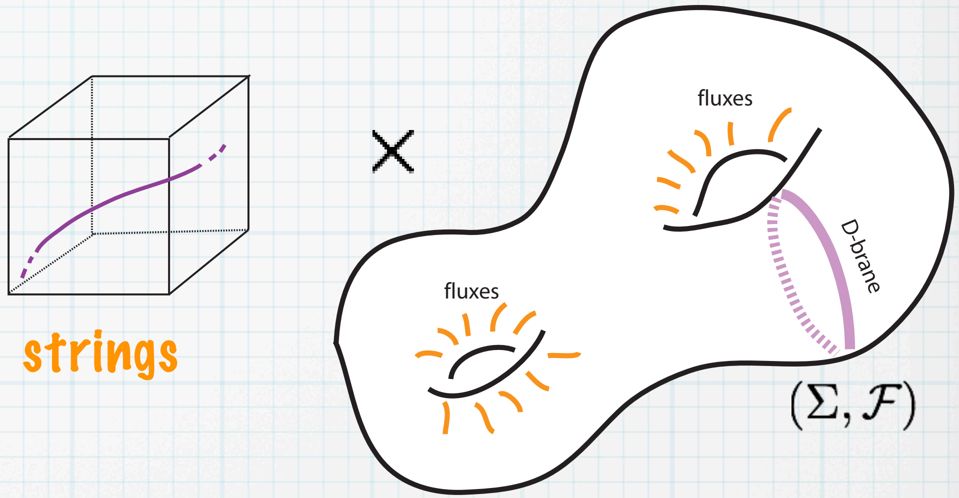
N=1 vacua and calibrations

L.M. & Smyth `05

- $e^{2A}\mathrm{Im}t$ is a generalized calibration for **D-strings**

$$\boxed{d_H(e^{2A}\mathrm{Im}t) = 0}$$

string BPSness



N=1 vacua and calibrations

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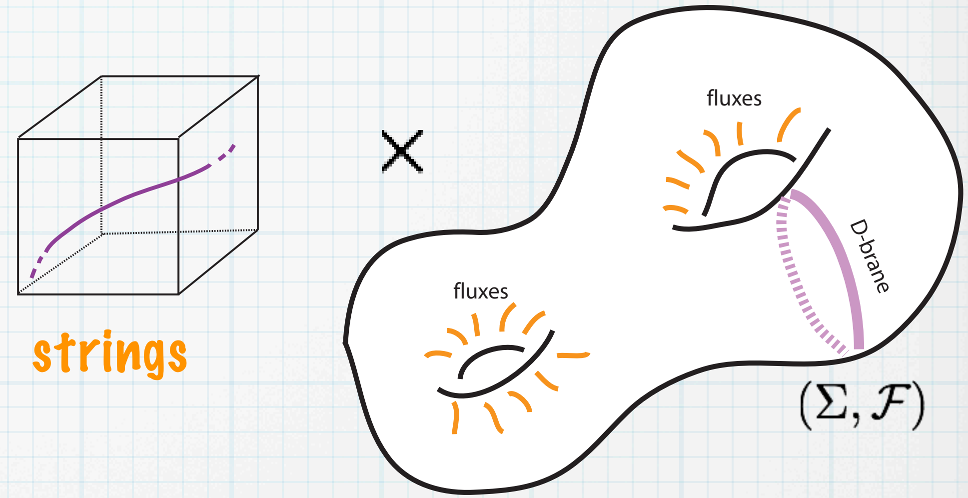
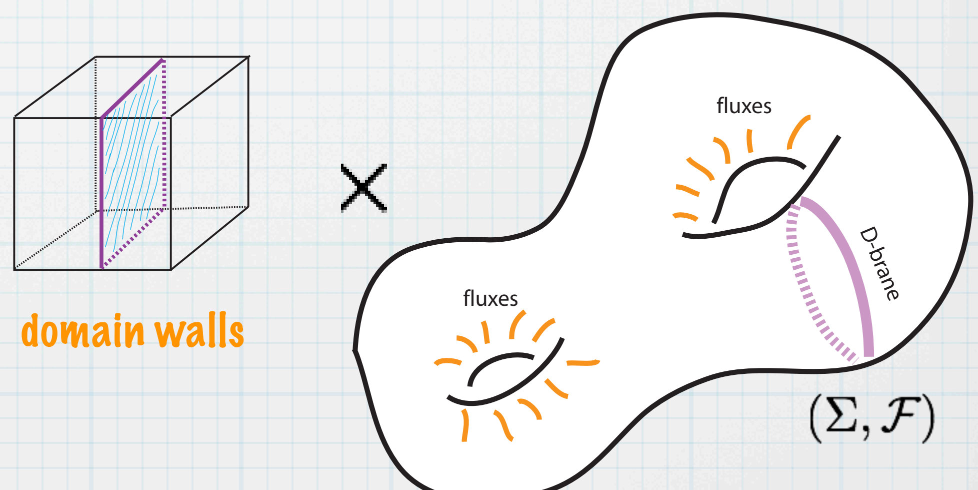
$$\boxed{d_H(e^{2A}\mathrm{Im}t) = 0}$$ *string BPSness*



strings

- $\mathrm{Re}(e^{i\theta}\mathcal{Z})$ is a generalized calibration for **domain walls**

$$\boxed{d_H \mathcal{Z} = 0}$$ *DW BPSness*



domain walls

Summarizing

- In N=1 compactifications (to flat $\mathbb{R}^{1,3}$) we have

Graña, Minasian, Petrini & Tomasiello `05 | L.M. & Smyth `05 | Koerber & L.M. `07

Equation	D-brane BPSness	4D SUGRA int.
$d_H(e^{4A}\mathrm{Re}\,t) = e^{4A} * F$	*gauge BPSness*	$\langle F_{\mathcal{Z}} \rangle = 0$
$d_H(e^{2A}\mathrm{Im}\,t) = 0$	*string BPSness*	$\langle \mathcal{D} \rangle = 0$
$d_H \mathcal{Z} = 0$	*DW BPSness*	$\langle F_{\mathcal{T}} \rangle = 0$

chiral fields: $\mathcal{T} = \mathrm{Re}t - iC_{\mathrm{RR}}$ (N=2 hypermult.)

$\mathcal{Z}$ (N=2 vect. mult.)

Pre-GG Example: type IIB warped CY

Graña & Polchinski `00;
Giddings, Kachru & Polchinski`01

- The internal metric is CY, up to warping:

$$\mathrm{d}s^2 = e^{2A(y)} g_4(x) + e^{-2A(y)} g_6^{\mathrm{CY}}(y)$$

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SUSY wCY and calibrations

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- **Gauge BPSness:**

  $\mathrm{d}_H(e^{4A}\mathrm{Re}\,t) = e^{4A} * F$

$$\bar{\partial}\tau = 0$$

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$$F_5 = *\mathrm{d}(e^{\Phi - 4A})$$

calibrated
D3 & D7 branes

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  $$\mathrm{d}J_{\mathrm{CY}} = 0$$

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  calibrated
  D5 & D7 branes

Calibrations, SUSY-breaking and 4D potential

Lüst, Marchesano, L.M. & Tsimpis`07

SUSY breaking?

- Pre-GG prototypical example: IIB wCY

Graña & Polchinski `00;

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  - N=1 and N=0 shear the same geometry

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  * flux induced SUSY-breaking: $H^{0,3} \neq 0$

- Many `phenomenological' models are based on these classical vacua

  e.g. *Kachru, Kallosh,, Linde & Trivedi `03; + MacAllister & Maldacena `03*

  Balasubramanian,, Berglund,, Conlon & Quevedo `05

~~SUSY~~ wCY and calibrations

- In this case: $t = e^{-\Phi}\exp\left(i\, e^{-2A} J_{\rm CY}\right)$, $\mathcal{Z} = \Omega_{\rm CY}$

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  calibrated D3 & D7 branes

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  $d_H(e^{2A}{\rm Im} t) = 0$

  $$dJ_{\rm CY} = 0$$
  $$H \wedge J_{\rm CY} = 0$$

  calibrated D3 & D7 branes

- DW (non)BPSness:

  $d_H \mathcal{Z} \neq 0$

  $$d\Omega_{\rm CY} = 0$$
  $$H \wedge \Omega_{\rm CY} \neq 0 \quad (H^{0,3} \neq 0)$$

  calibrated D5 & ~~D7~~ branes

Generalized SUSY-breaking

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- We are led to consider backgrounds that fulfill **gauge and string BPSness**, but **not DW BPSness**

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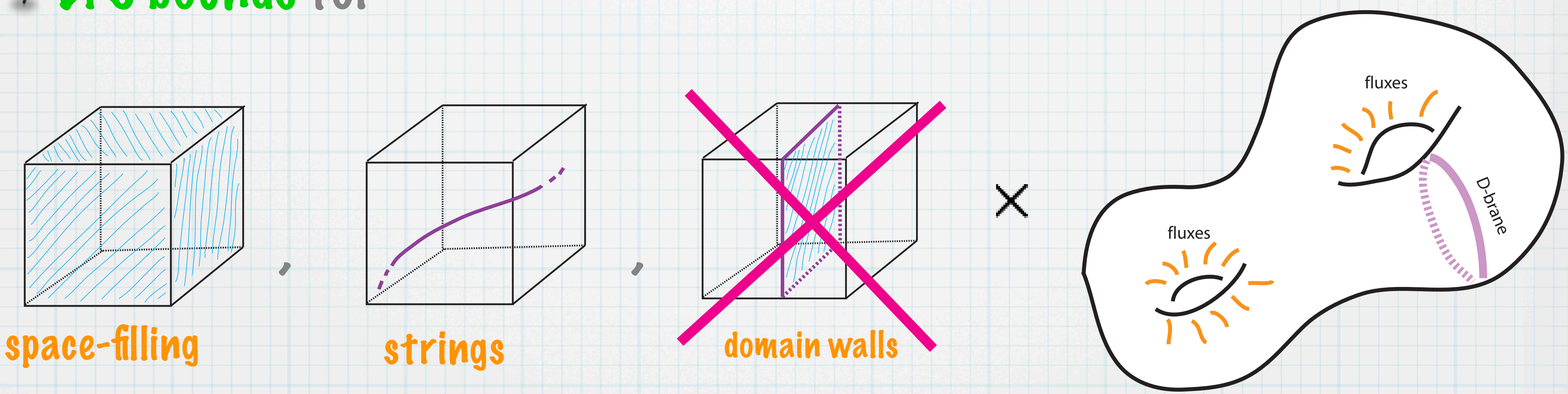
- BPS bounds for



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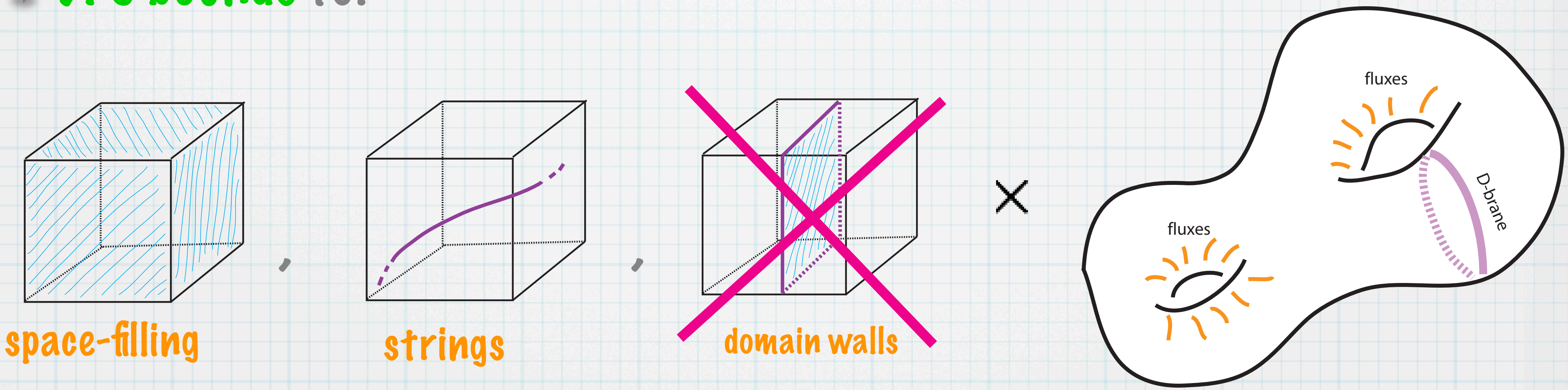

DWSB backgrounds

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DWSB backgrounds

Generalized SUSY-breaking

- In wCY, the DWSB has rather specific form

$$d_H \mathcal{Z} = d_H \Omega_{\rm CY} = H \wedge \Omega_{\rm CY} \neq 0 \quad \rightarrow$$

crucial to solve
10D e.o.m.

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crucial to solve 10D e.o.m.

Can we analogously restrict the generalized DWSB?

$$\mathrm{d}_H \mathcal{Z} = \,?$$

1-parameter DWSB

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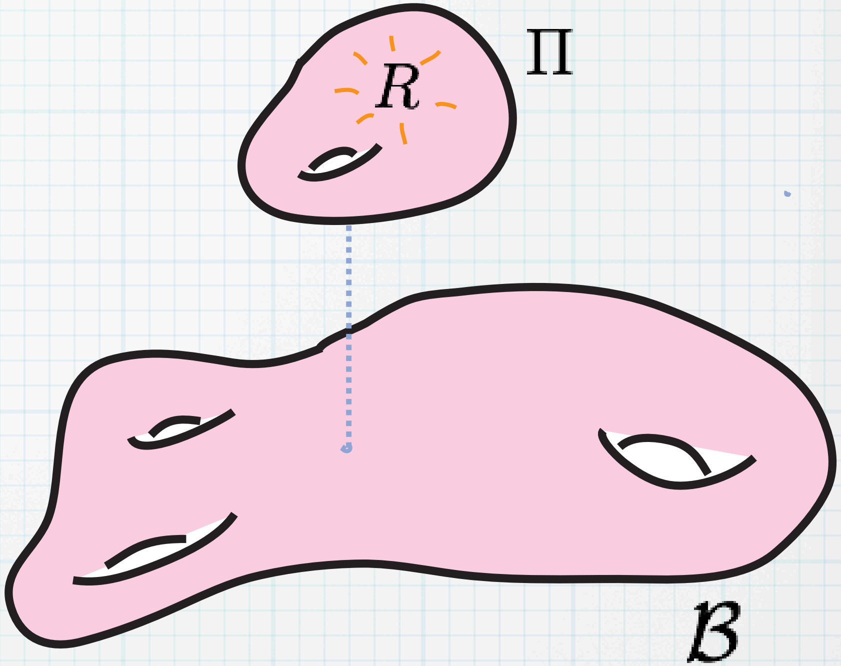
- Take generalized fibration (Π, R)
  (Dirac structure)

$$\mathrm{d}R = H|_{\Pi}$$



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spanned by mobile D-branes

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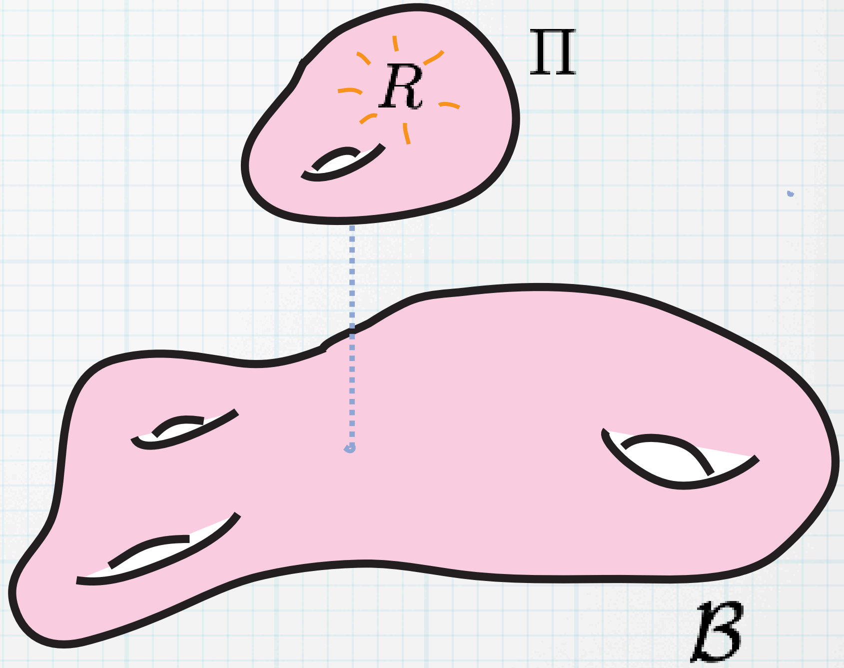

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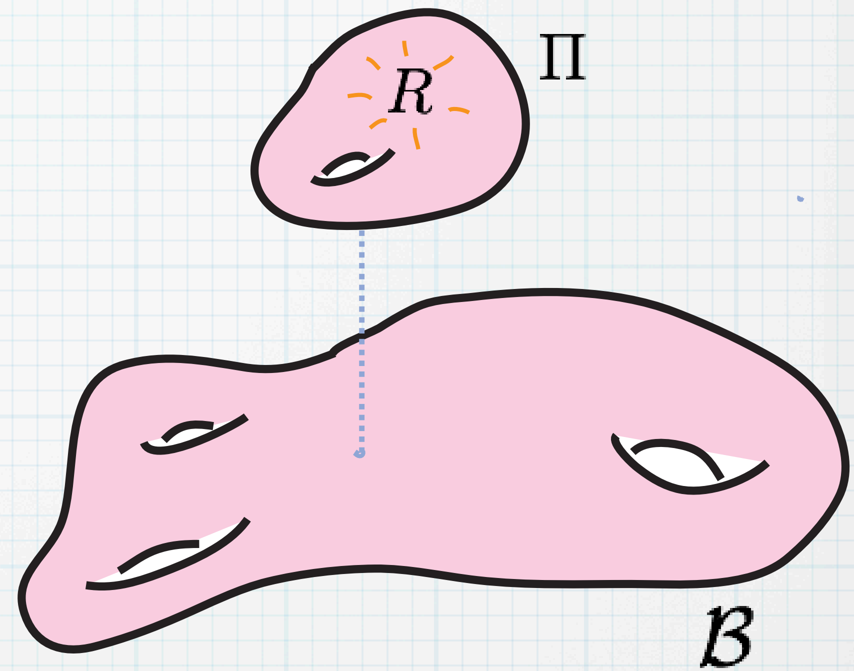
  $$\mathrm{d}R = H|_{\Pi}$$

- Consider DWSB of the form

$$\mathrm{d}_H \mathcal{Z} = r\tilde{j}_{(\Pi,R)}$$

$$\simeq e^{-R}\mathrm{dVol}_{\mathcal{B}}$$



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SUSY-breaking parameter

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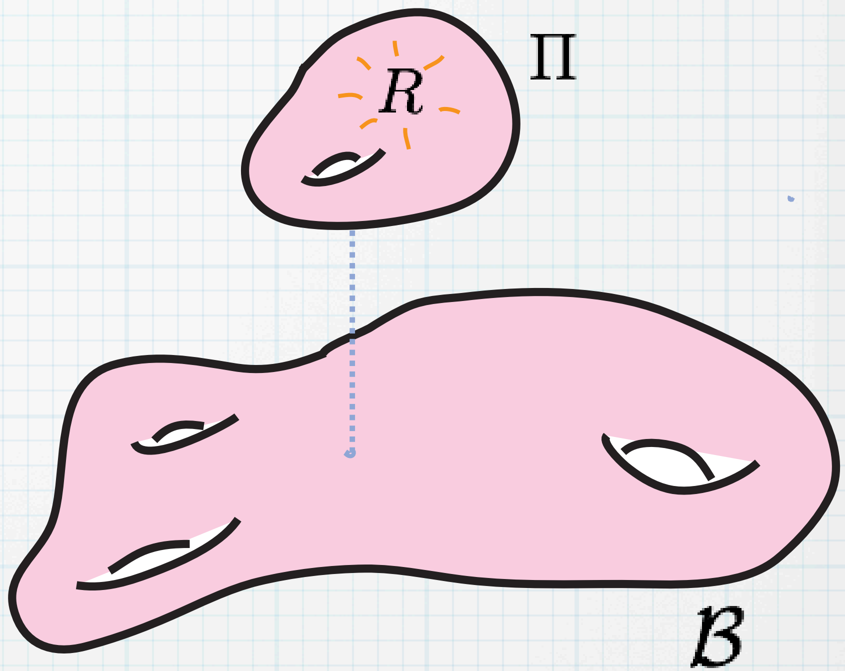

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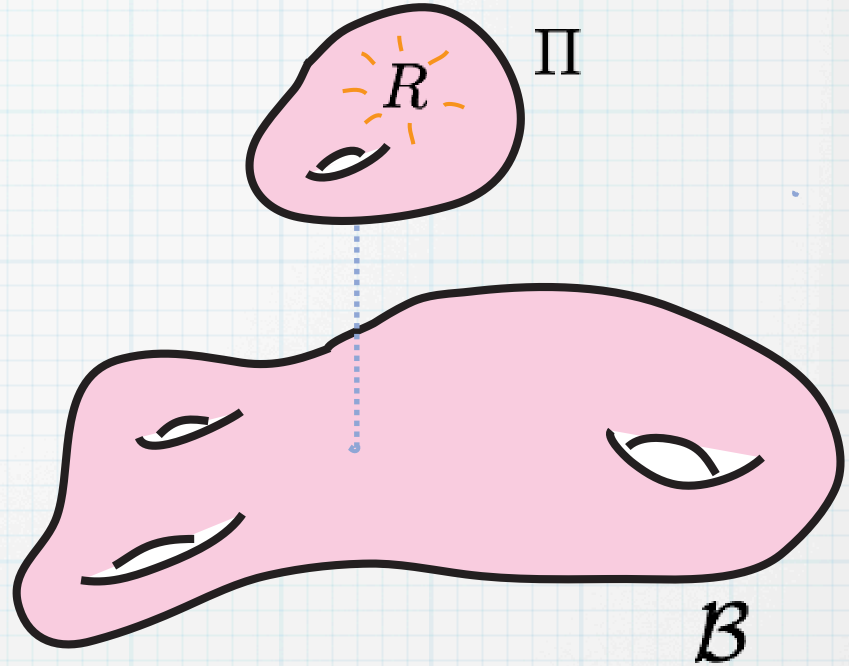

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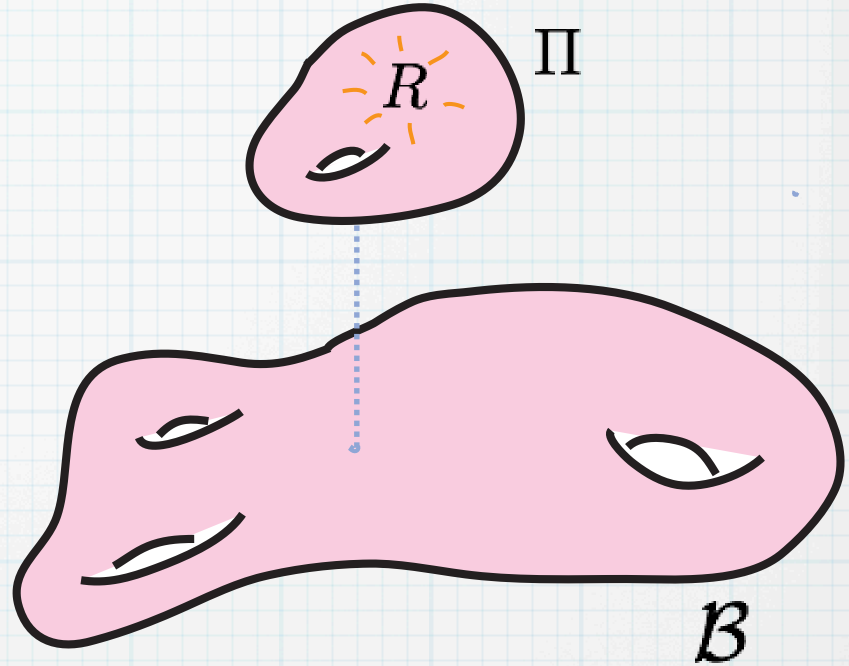

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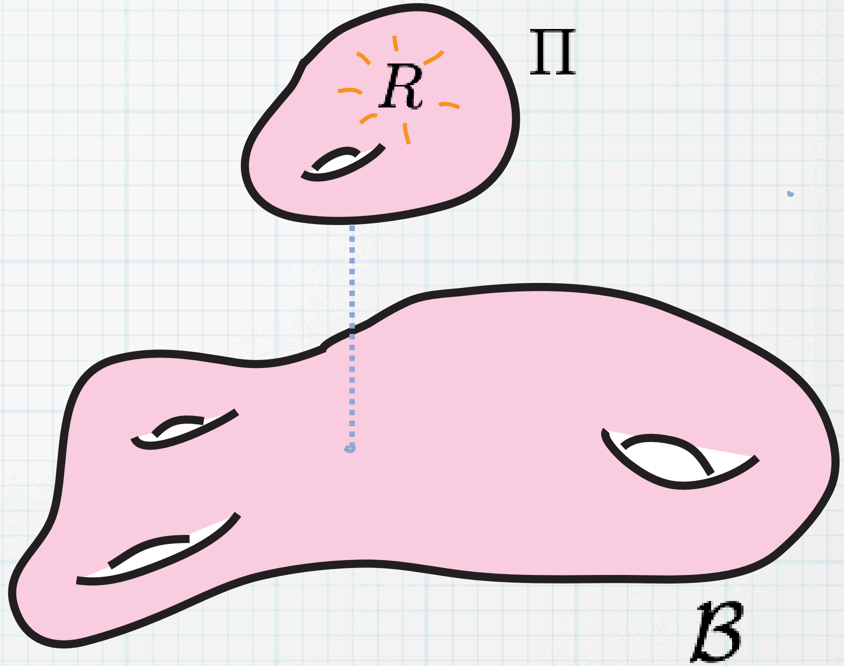

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  spanned by mobile D3

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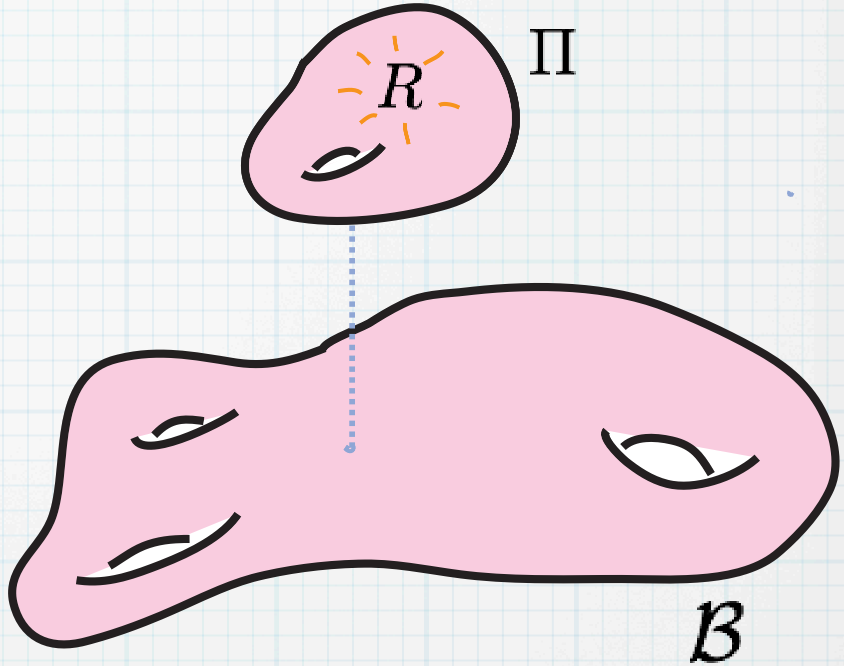

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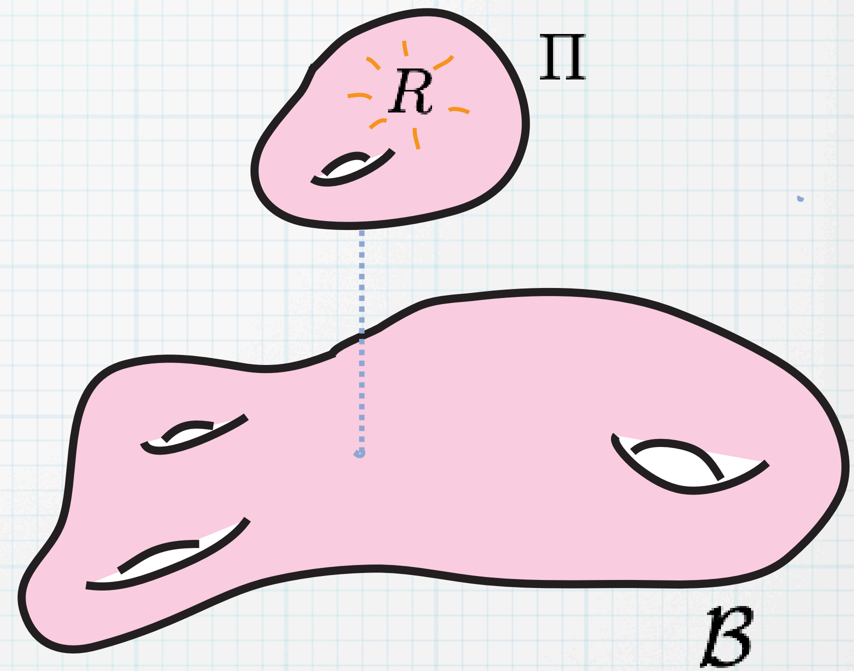

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  $\mathcal{B} = M$

  $\tilde{\jmath} \sim \mathrm{dVol}_6$

  $\Rightarrow$ $\mathrm{d}_H \mathcal{Z} = \mathrm{d}_H \Omega_{\mathrm{CY}} \simeq r\,\mathrm{dVol}_6$

  $\Rightarrow$ $\mathrm{d}\Omega_{\mathrm{CY}} = 0$, $r \simeq H^{0,3}$

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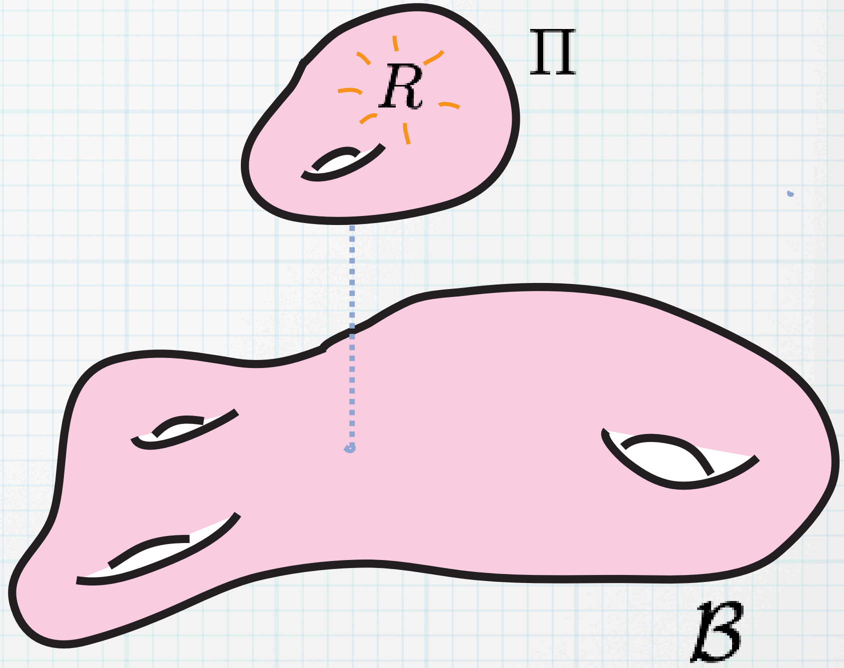

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How to check that 10D e.o.m. are satisfied?

Which conditions must (Π, R) fullfill?

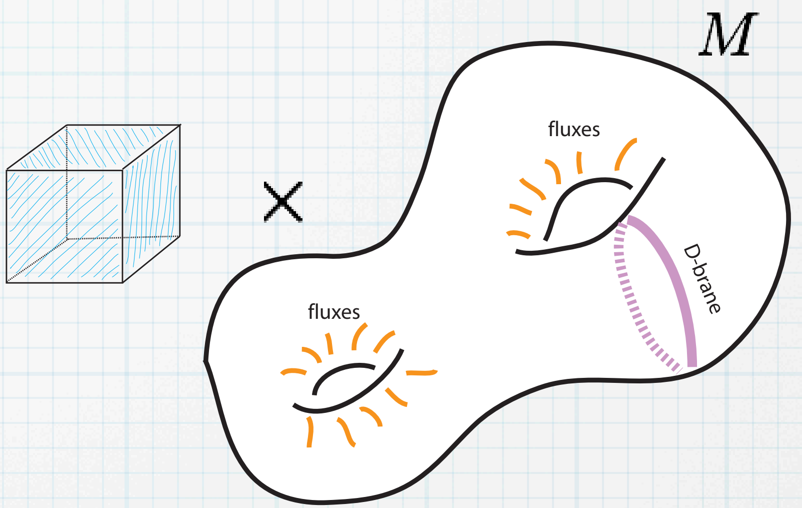
10D e.o.m. from 4D potential

- General configurations of the form

$$\mathrm{d}s^2 = e^{2A(y)} g_4(x) + g_6(y)$$

plus H , Φ , F ,

D-branes & orientifolds



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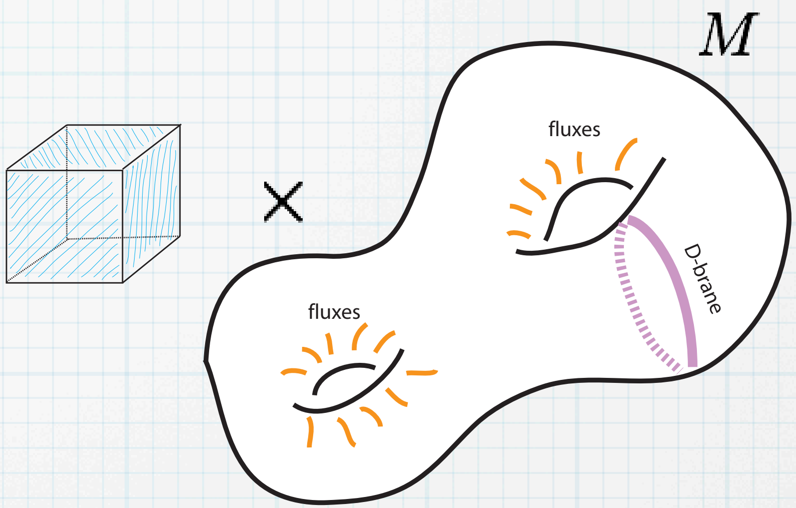

- The full set of 10D e.o.m. can be obtained from

$$F_{\mathrm{el}} = *F$$

$$V = \int_M \mathrm{dVol}_6\, e^{4A} \left\{ e^{-2\Phi} \left[-R_6 + \frac{1}{2} H^2 - 4(\mathrm{d}\Phi)^2 + 8\nabla^2 A + 20(\mathrm{d}A)^2 \right] - \frac{1}{2} F_{\mathrm{el}}^2 \right\}$$

$$+ \sum_{i \in \mathrm{loc.\ sources}} \tau_i \left(\int_{\Sigma_i} e^{4A-\Phi} \sqrt{\det(g|_{\Sigma_i} + \mathcal{F}_i)} - \int_{\Sigma_i} C^{\mathrm{el}}|_{\Sigma_i} \wedge e^{\mathcal{F}_i} \right)$$

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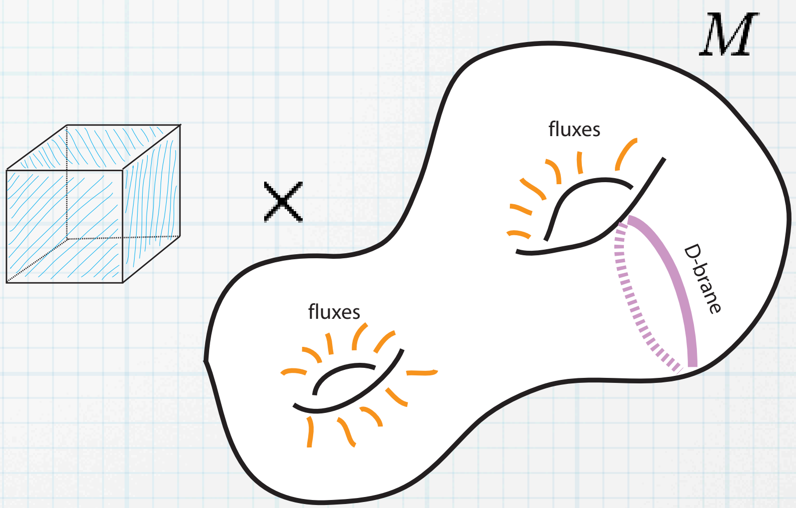

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We need to express the potential in terms of t and z

Potential and pure spinors

We found, schematically (see also Cassani `08)

$$
\begin{aligned}
V \quad = \quad & V_{\text{D-branes}} - V_{\text{D-branes}}^{\text{BPS}} \\
& + \int_M [\mathrm{d}_H(e^{4A}\mathrm{Re}t) - e^{4A} * F]^2 \\
& + \int_M [\mathrm{d}_H(e^{2A}\mathrm{Im}t)]^2 \\
& + \int_M |\mathrm{d}_H \mathcal{Z}|^2 \\
& - \int_M |\langle t, \mathrm{d}_H \mathcal{Z}\rangle|^2 \\
& - \quad (\ldots)
\end{aligned}
$$

Potential and pure spinors

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$$
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& + \int_M [\mathrm{d}_H(e^{2A}\mathrm{Im}t)]^2 \quad \gtrsim 0 \\
& + \int_M |\mathrm{d}_H \mathcal{Z}|^2 \quad \gtrsim 0 \\
& - \int_M |\langle t, \mathrm{d}_H \mathcal{Z}\rangle|^2 \quad \lesssim 0 \\
& - \quad (\ldots) \quad \lesssim 0
\end{aligned}
$$

Potential and pure spinors

We found, schematically (see also Cassani `08)

$$V = V_{\text{D-branes}} - V^{\text{BPS}}_{\text{D-branes}} \gtrsim 0$$ (D-brane BPS bound)

$$+ \int_M [\mathrm{d}_H(e^{4A}\mathrm{Re}t) - e^{4A} * F]^2 \gtrsim 0$$ (gauge BPSness)^2 $\sim |F_{\mathcal{Z}}|^2$

$$+ \int_M [\mathrm{d}_H(e^{2A}\mathrm{Im}t)]^2 \gtrsim 0$$ (string BPSness)^2 $\sim \mathcal{D}^2$

$$+ \int_M |\mathrm{d}_H \mathcal{Z}|^2 \gtrsim 0$$ (DW BPSness)^2 $\sim |F_{\mathcal{T}}|^2$

$$- \int_M |\langle t, \mathrm{d}_H \mathcal{Z}\rangle|^2 \lesssim 0$$ (DW BPSness)^2 $\sim |F_{\mathcal{T}}|^2$

$$- \quad (\ldots) \lesssim 0$$ (string + DW BPSness)^2 $\sim |\mathcal{D} + F_{\mathcal{T}}|^2$

Potential for 1-param. DWSB

$$V = V_{\text{D-branes}} - V^{\text{BPS}}_{\text{D-branes}} \gtrsim 0$$

$$+ \int_M [\mathrm{d}_H(e^{4A}\mathrm{Re}t) - e^{4A} * F]^2 \gtrsim 0$$

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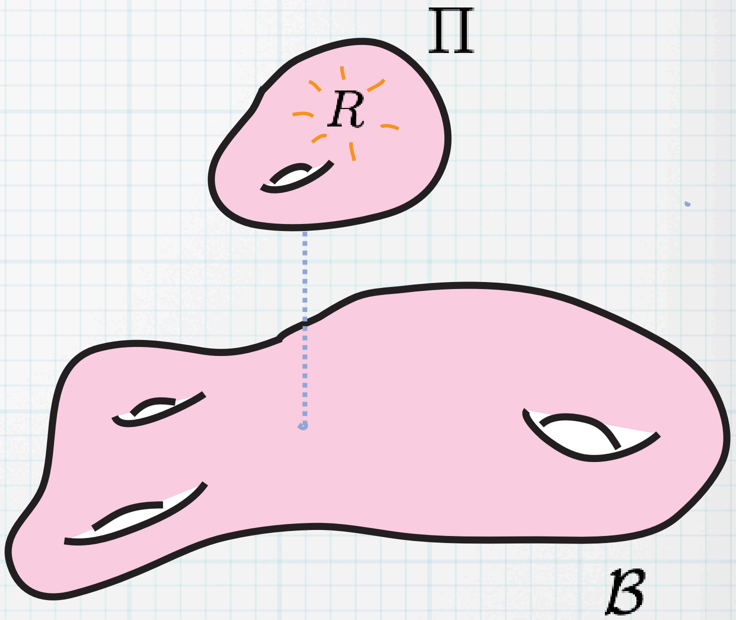
$$+ \int_M r\left[\mathcal{E}_{\text{DBI}}(\Pi, R) - \mathcal{E}^{\text{BPS}}_{\text{DBI}}(\Pi, R)\right] \gtrsim 0$$



$$\mathrm{d}_H \mathcal{Z} \simeq r\, e^{-R} \mathrm{dVol}_{\mathcal{B}}$$

Potential for 1-param. DWSB

calibrated D-branes

$$V = \cancel{V_{\text{D-branes}} - V^{\text{BPS}}_{\text{D-branes}}} \quad \geq 0$$

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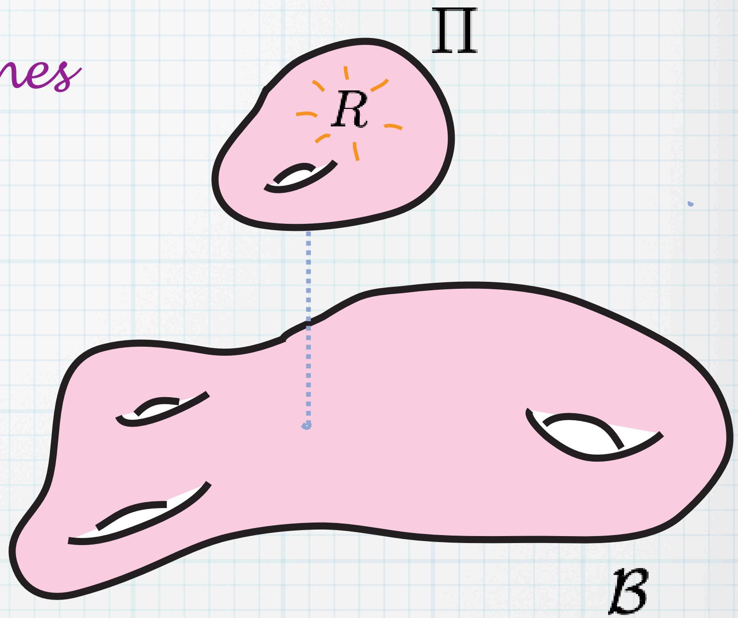

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Potential for 1-param. DWSB

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gauge BPSness ($\langle F_{\mathcal{Z}} \rangle \simeq 0$)

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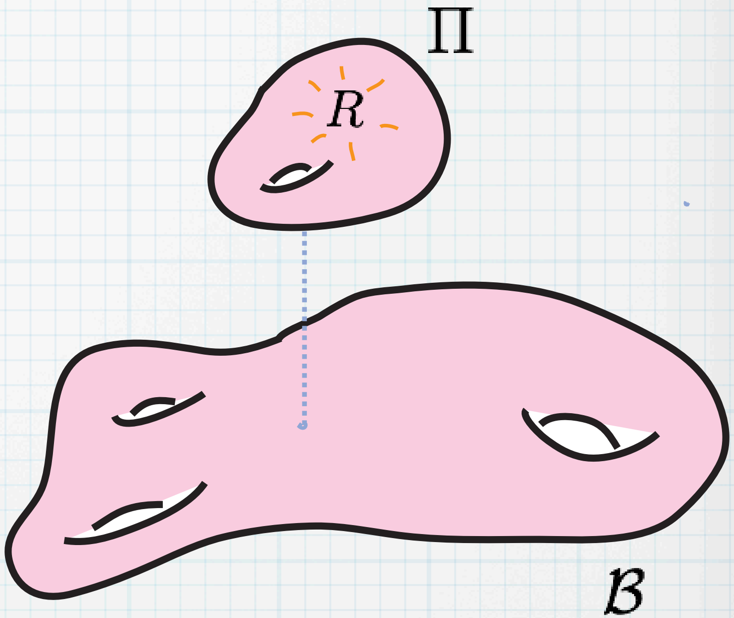

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string BPSness $(\langle \mathcal{D} \rangle = 0)$

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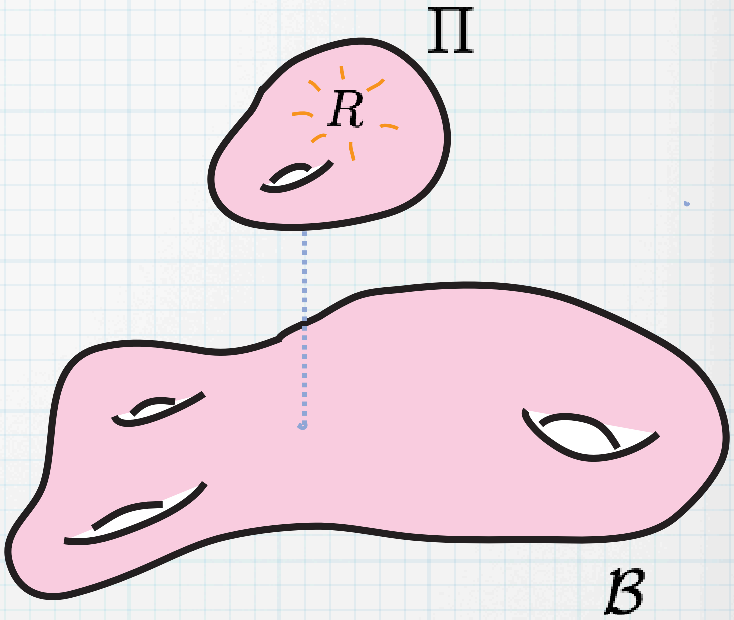

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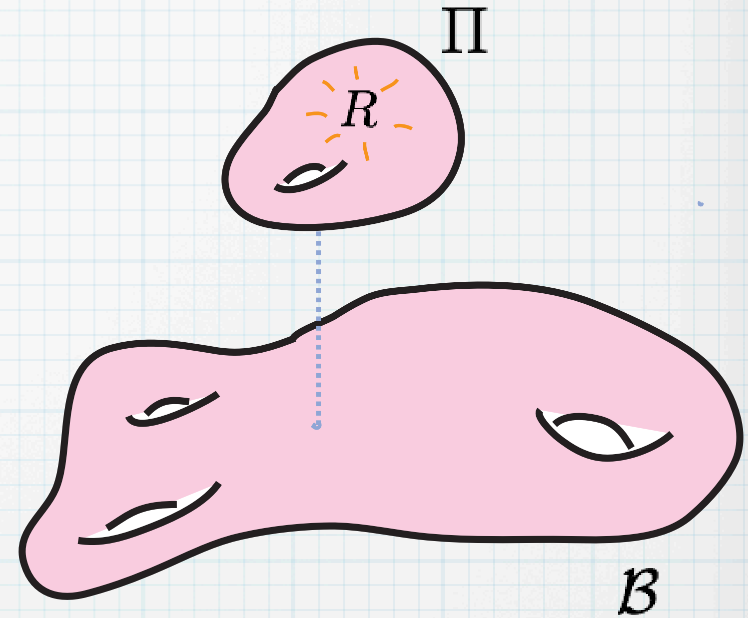

$$\mathrm{d}_H \mathcal{Z} \simeq r\, e^{-R} \mathrm{dVol}_{\mathcal{B}}$$

calibrated generalized fibration (Π, R)

$(\langle F_T \rangle \sim r)$

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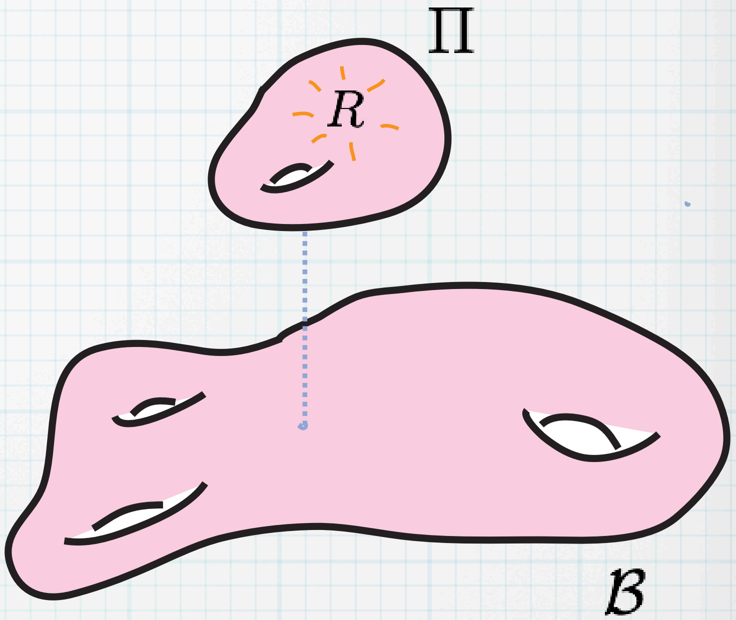

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calibrated generalized fibration (Π, R)

$(\langle F_T \rangle \sim r)$

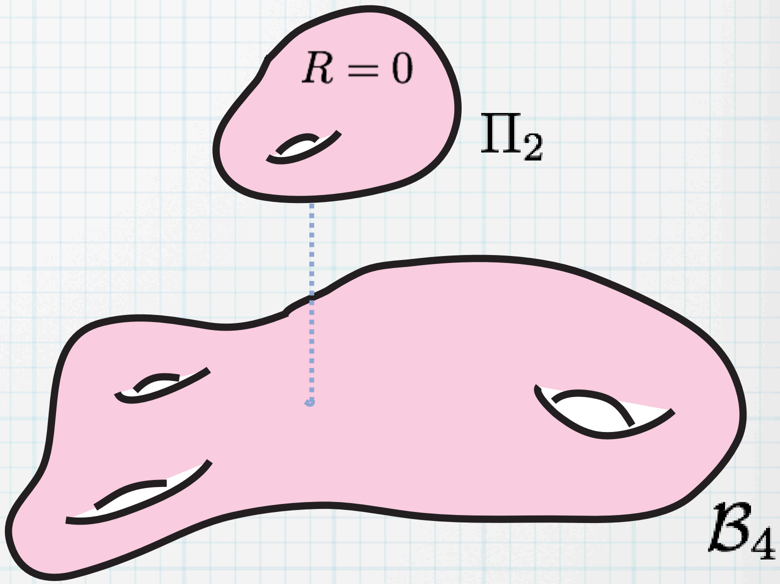
r -dependence disappears:

no-scale structure

An example

Example: SU(3)-structure with D5/O5

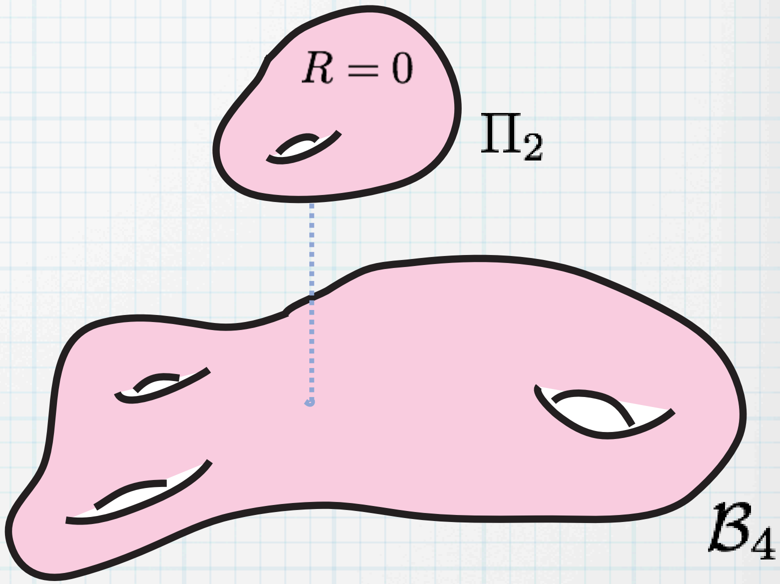
[cfr. Camara & Graña `07]



Example: SU(3)-structure with D5/O5

[cfr. Camara & Graña `07]

- In this case: $t = -ie^{-\Phi}e^{iJ}$, $\mathcal{Z} = e^{3A-\Phi}\Omega$



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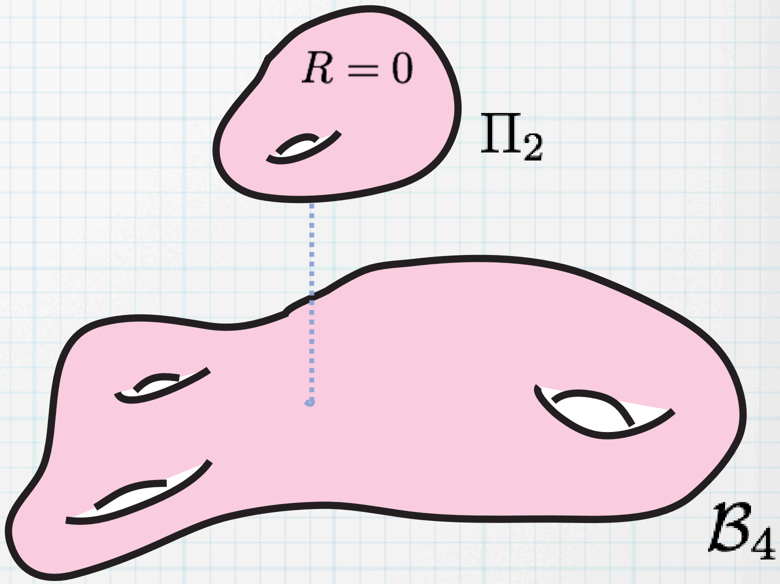
$$H = F_1 = F_5 = 0$$

$$\mathrm{d}J \wedge J = 0 \qquad e^{2A-\Phi} = \text{const.}$$

$$(1 + i*)\Big[F_3 + i\, e^{-2\Phi}\mathrm{d}(e^{\Phi}J)\Big] = 0 \quad \text{ISD}$$

$$\longrightarrow \quad \mathrm{d}J \neq 0 \quad \text{non-Kähler}$$



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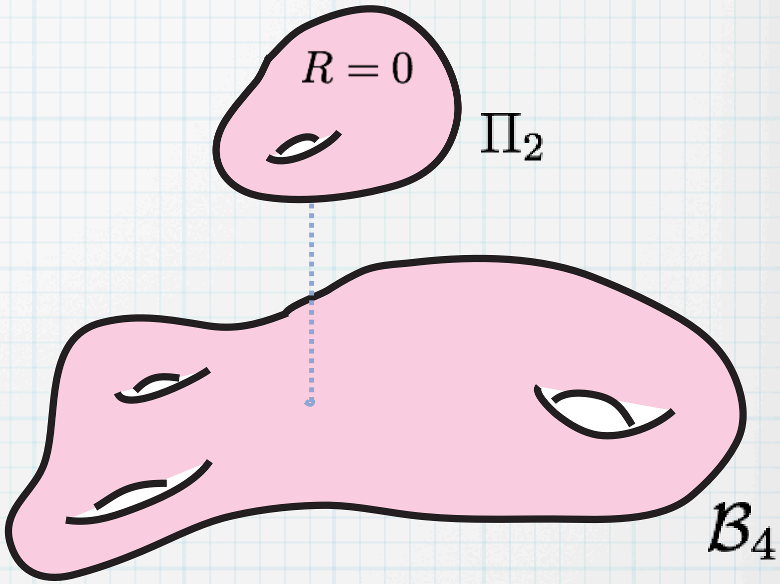

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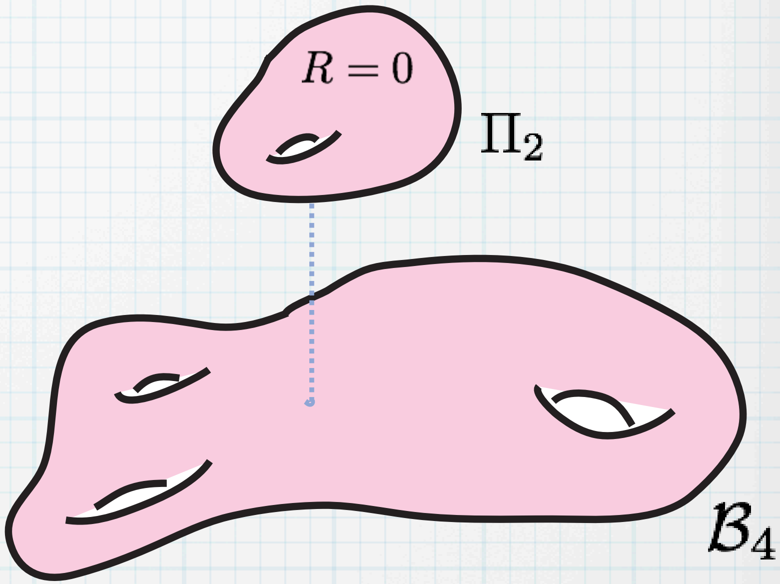

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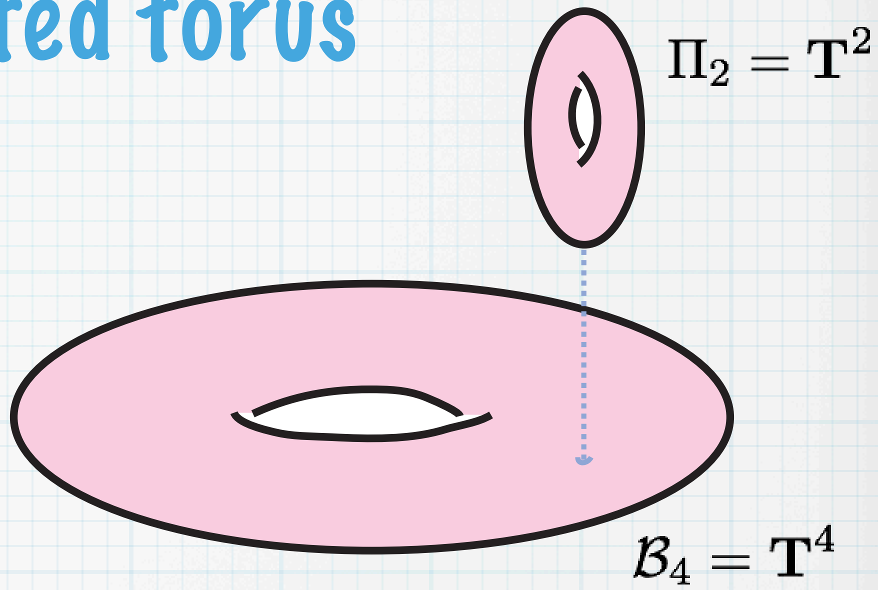
- Notice that: $\mathrm{d}\Omega \neq 0$ → non-integrable complex structure if $r \neq 0$

Explicit example on twisted torus



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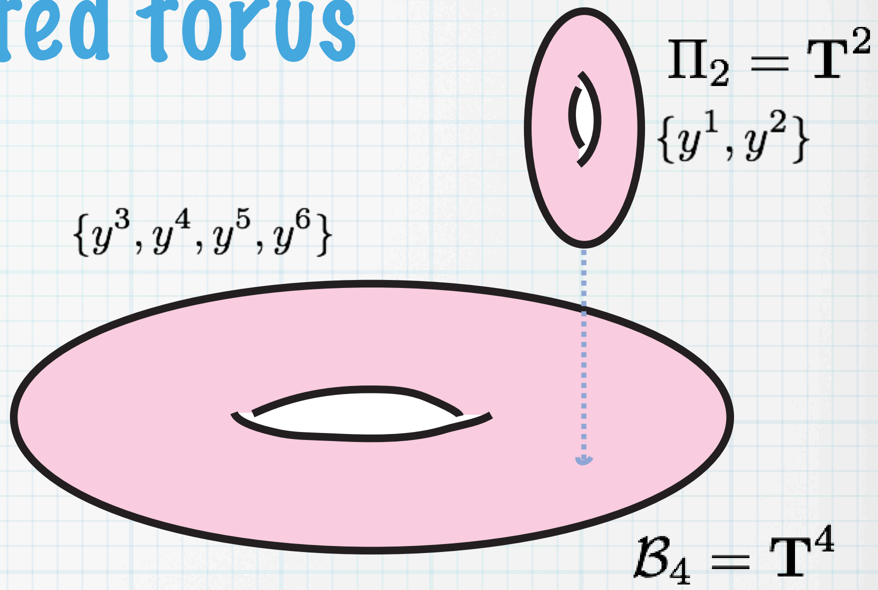
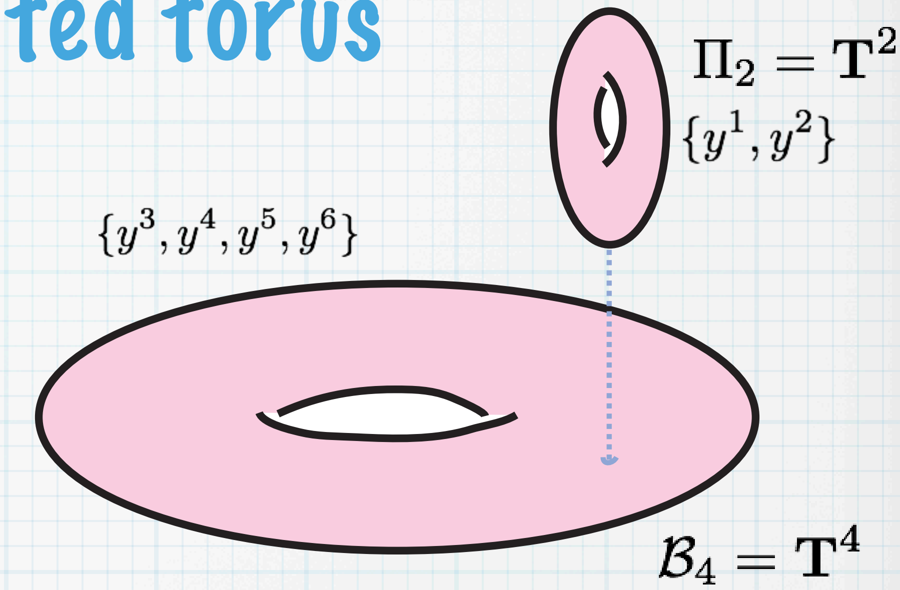


Explicit example on twisted torus

- Twisted periodicity in 3rd direction:

$$\{y^1, y^3, y^5\} \simeq \{y^1 + k\, y^5, y^3 + 1, y^5\}$$



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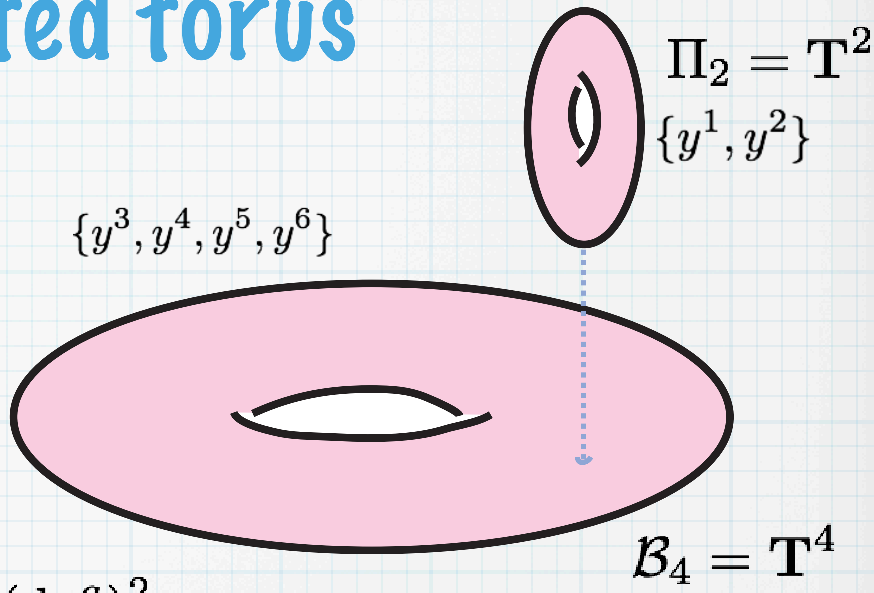

- Metric and fields

$$\mathrm{d}s_6^2 = e^{2A}[(\mathrm{d}y^1 - ky^3\mathrm{d}y^5)^2 + (\mathrm{d}y^2)^2] + e^{-2A}\sum_{a=3}^{6}(\mathrm{d}y^a)^2$$

$$e^{\Phi} = g_s e^{2A} \quad , \quad F_3 = -g_s^{-1} *_{\mathbf{T}^4} e^{-4A} - N(\mathrm{d}y^1 - ky^3\mathrm{d}y^5) \wedge \mathrm{d}y^4 \wedge \mathrm{d}y^6$$

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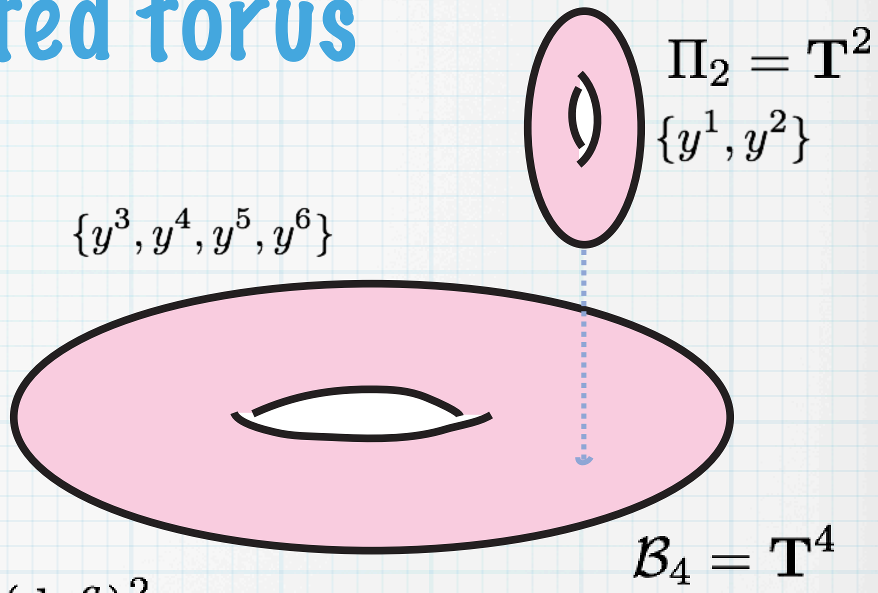

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- SU(3)-structure

$$J = e^{2A}(\mathrm{d}y^1 - ky^3\mathrm{d}y^5)\wedge \mathrm{d}y^2 + e^{-2A}(\mathrm{d}y^3\wedge \mathrm{d}y^4 + \mathrm{d}y^5 \wedge \mathrm{d}y^6)$$

$$\Omega = e^{-A}[(\mathrm{d}y^1 - ky^3\mathrm{d}y^5) + i\mathrm{d}y^2]\wedge(\mathrm{d}y^3 + i\mathrm{d}y^4)\wedge(\mathrm{d}y^5 + i\mathrm{d}y^6)$$

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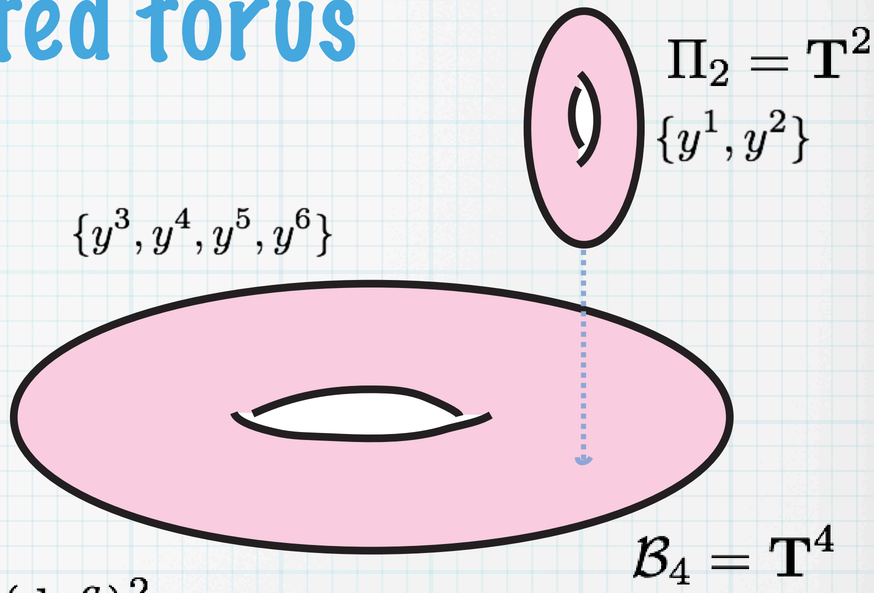

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- DWSB $\quad \mathrm{d}(e^A\Omega) = -k\,\mathrm{d}y^3\wedge\mathrm{d}y^4\wedge\mathrm{d}y^5\wedge\mathrm{d}y^6 \neq 0$ → $r \sim k$ twist-induced DWSB

Conclusion

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- ~~SUSY~~ →
  - ~~integrable GC~~ but still calibration structures
  - less clear N=1 4D structures