Class assignment 4: Exercises in set theory

Rules: This is a test to be done in class during your monitor session. Please solve it in class and give to your monitor. Your solutions will have little influence on the final grade. If you use some result, please state it and (unless it's trivial) explain the proof.

Exercise 4.1. Let ω_1 be the smallest ordinal which is not countable. Prove that any non-countable set contains a subset which is equinumerous with ω_1.

Exercise 4.2. Let $A \subset B$ be a subset of a well ordered poset B. Prove that A is well ordered, or find a counterexample.

Exercise 4.3. Let A, B be two ordinals (classes of isomorphism of well ordered posets). Suppose that there exist a map $A \longrightarrow B$ preserving the order, and a map $B \longrightarrow A$ preserving the order. Prove that A and B are isomorphic.

Exercise 4.4. An ordinal A is called **additively indecomposable** if for any ordinals B, C such that $A = B + C$, one has $C = A$. Prove that there exists a non-empty additively indecomposable ordinal.

Exercise 4.5 (2 points). Let ω_1 be the smallest ordinal which is not countable. Prove that its square ω_1^2 is equinumerous to ω_1.