Class assignment: Rational functions on algebraic varieties

Rules: This is a test to be done in class during your monitor session. Please solve it in class and give to your monitor Renan Santos. Your solutions will have little influence on the final grade. If you use some result, please state it and (unless it's trivial) explain the proof.

Exercise 13.1. Let f be a continuous rational function on $\mathbb{C}^n$. Prove that f is polynomial.

Exercise 13.2 (2 points). Let $M \subset \mathbb{C}^n$ be an irreducible algebraic variety, and f a continuous rational function on M. Prove that f is regular, or find a counterexample.

Exercise 13.3 (2 points). Let $f: \ M_1 \longrightarrow M_2$ be a morphism of affine subvarieties. Assume that f is a homeomorphism. Prove that f is an isomorphism, or find a counterexample.

Exercise 13.4 (2 points). Let $Z \subset \mathbb{C}^2$ be a curve given by an equation $x^2 + y^2 = 1$. Construct a birational map from Z to $\mathbb{C}$.

Exercise 13.5. Let $F: \ \mathbb{C}^3 \longrightarrow \mathbb{C}^3$ be a morphism defined by the formula $F(x, y, z) = (xy, xz, xyz)$.

a. Prove that this map is birational.

b. Prove that it is not an isomorphism.