# Home assignment 8: Equivalence of categories

**Rules:** This is a class assignment for the next week. Please solve as many exercises as you can, bring me what you can before the Wednesday week after. Wednesdays 17:00 we will discuss the solutions in a monitor session. Exercises with [*] are extra hard and not necessary to follow the rest. Exercises with [!] are non-trivial, fundamental and necessary for further work.

**Exercise 8.1.** Prove that the category of all finitely generated $\mathbb{C}[t, t^{-1}]$-modules is equivalent to the category of finitely generated representations of $\mathbb{Z}$.

**Exercise 8.2.** Let $\mathcal{C}_1$ be the category of all finite-dimensional representations of $\mathbb{Z}/2\mathbb{Z}$ over $\mathbb{C}$, and $\mathcal{C}_2$ the category of all representations (finite and infinite-dimensional). Are the categories $\mathcal{C}_1$ and $\mathcal{C}_2$ equivalent?

**Exercise 8.3.** Let $\mathcal{C}_1$ be the category of all finite-dimensional representations of $\mathbb{Z}/2\mathbb{Z}$ over $\mathbb{C}$, and $\mathcal{C}_2$ the category of all finite-dimensional vector spaces over $\mathbb{C}$. Are the categories $\mathcal{C}_1$ and $\mathcal{C}_2$ equivalent?

**Exercise 8.4.** Let $\mathcal{C}_1$ be the category of all finite-dimensional representations of $\mathbb{Z}/2\mathbb{Z}$ over $\mathbb{C}$, and $\mathcal{C}_2$ the category of all finite-dimensional representations of the symmetric group $S_3$ over $\mathbb{C}$. Are the categories $\mathcal{C}_1$ and $\mathcal{C}_2$ equivalent?

**Exercise 8.5 (*).** Let $\mathcal{C}_1$ be the category of all finite-dimensional representations of $\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$ over $\mathbb{C}$, and $\mathcal{C}_2$ the category of all finite-dimensional representations of $\mathbb{Z}/4\mathbb{Z}$ over $\mathbb{C}$. Are the categories $\mathcal{C}_1$ and $\mathcal{C}_2$ equivalent?

**Exercise 8.6.** Let $\mathcal{C}_1$, $\mathcal{C}_2$ be the categories of finite-dimensional vector spaces over $\mathbb{R}$ and $\mathbb{C}$. Are the categories $\mathcal{C}_1$ and $\mathcal{C}_2$ equivalent?

**Exercise 8.7 (*).** Prove that the category of finite groups is not equivalent to the category of finite abelian groups.