Class test 2: Holomorphic functions and complex structures

Exercise 2.1. Let f be a holomorphic function on disk $\Delta \subset \mathbb{C}$ which is continuoulsly extended to the boundary $\partial\Delta$. Suppose that $f\big|_{\partial\Delta}$ takes values in $\mathbb{R}$. Prove that f is constant, or find a counterexample.

Exercise 2.2. Let $Z \subset \mathbb{C}$ be a finite subset, and f a bounded holomorphic function on $\mathbb{C}\backslash Z$. Prove that f is a constant, or find a counterexample.

Definition 2.1. A function f is **antiderivative** of g if $f' = g$.

Exercise 2.3. Let $P(t) \in \mathbb{R}[t]$ be a polynomial of degree 3 with 3 different roots on $\mathbb{R}$, and $f := \sqrt{P(t)}$. Prove that f exists and is holomorphic on upper halfplane, and its antiderivative maps the upper halfplane to a rectangle.

Exercise 2.4. Let $\rho \in \Lambda^2(V)$ be a 2-form on a real vector space V equipped with a complex structure operator $I \in \operatorname{End}(V)$, $I^2 = -\mathsf{Id}$. Assume that $\rho(x, Iy) = \rho(Ix, y)$ for all $x, y \in V$. Prove that ρ is a real part of a $(2,0)$-form.