

Class test 3: almost complex structures on manifolds

Exercise 3.1. Let M be an almost complex manifold. Prove that M is orientable.

Definition 3.1. Riemannian metric on a manifold M is a smooth tensor $g \in \text{Sym}^2(T^*M)$ which defines a positive definite bilinear symmetric form in each tangent space T_xM .

Exercise 3.2. Prove that any manifold admits a Riemannian metric.

Exercise 3.3. Let M be a 2-dimensional orientable manifold. Prove that M admits an almost complex structure.

Exercise 3.4. Let M be an almost complex manifold. Prove that M admits a non-degenerate 2-form.