

## Complex varieties 4, written test: Germs of complex varieties

**Rules:** This is a written test for the next week. Please write your solutions and bring me no later than the Monday class the week after. We shall discuss this test in class afterwards.

**Exercise 4.1.** Let  $R$  be the ring of germs of a variety  $X \subset \mathbb{C}^3$  given by an equation  $x^2 = yz$  in zero; equivalently,  $R := \mathcal{O}_3/(x^2 - yz)$ .

- a. Prove that the images of  $x$ ,  $y$  and  $z$  in  $R$  cannot be decomposed onto a product of non-invertible functions.
- b. Prove that  $\frac{x}{y}$  is not holomorphic in  $R$ .
- c. Prove that  $R$  is not factorial.

**Exercise 4.2.** Let  $A, B \subset \mathbb{C}^n$  be complex subvarieties, and  $J_A, J_B \subset \mathcal{O}_{\mathbb{C}^n}$  their ideals. Find an example of  $A, B$  such that  $J_{A \cap B} \not\supseteq J_A + J_B$ .

**Exercise 4.3.** Determine the singular set of the germs of complex varieties defined by the following ideals.

- a.  $(xy, yz, zx) \subset \mathcal{O}_{\mathbb{C}^3}$
- b.  $(xy, yz) \subset \mathcal{O}_{\mathbb{C}^3}$