Teoria Ergódica Diferenciável, assignment 9: orthogonal group

Rules: This is a home assignment for November 10. Please bring me the solutions no later than November 17.

Exercise 9.1. Let g be a quadratic form of signature (1,1) on $V = \mathbb{R}^2$, and $P : V \longrightarrow V$ a map preserving g (that is, satisfying $g(v) = g(P(v))$ for all $v \in V$). Assume that P belongs to connected component of $SO(1,1)$. Prove that all eigenvalues of P are real.

Exercise 9.2. Let g be a quadratic form of signature (1,2) on $V = \mathbb{R}^2$, and $P : V \longrightarrow V$ a map preserving g. Prove that P has one eigenvalue which is equal to ± 1.

Exercise 9.3. Let $V = \mathbb{C}^2$, and g, h two complex linear non-degenerate bilinear symmetric forms on V. Prove that there exists a basis x_1, x_2 which is orthogonal with respect to g, h, or find a counterexample.

Exercise 9.4. Let $V = \mathbb{R}^2$, and g, h two non-degenerate bilinear symmetric forms of signature (1,1) on V. Prove that there exists a basis x_1, x_2 in the complexification $V \otimes_{\mathbb{R}} \mathbb{C} = \mathbb{C}^2$ which is orthogonal with respect to g, h, or find a counterexample.