Topologia das Variedades: class exercise 1

Exercise 1.1. Let $TS^2 \oplus \mathbb{R}$ be a direct sum of a tangent bundle TS^2 and a trivial 1-dimensional bundle. Is the bundle $TS^2 \oplus \mathbb{R}$ trivial?

Exercise 1.2. Let M be a simply connected manifold. Prove that any real rank 1 bundle on M is trivial.

Exercise 1.3. Let $M_1 \xrightarrow{\phi} M$ be a surjective, smooth map without critical points, M, M_1 compact manifolds, and B a non-trivial bundle on M. Can the pullback bundle $\phi^* B$ be trivial?

Exercise 1.4. Construct a non-trivial rank 2 vector bundle which does not have any non-trivial sub-bundles.

Exercise 1.5. Let $V = \mathbb{R}^4$, and $\alpha \in \Lambda^2 V^*$. Assume that $\alpha \wedge \alpha \in \Lambda^4 V^*$ is non-zero. Prove that α is a symplectic 2-form.