# Topologia das Variedades: class exercise 4

**Definition 4.1.** Unitary group $U(n)$ is the group of complex isometries of a complex Hermitian space $\mathbb{C}^n$. **Special unitary** group $SU(n)$ is all $g \in U(n)$ with determinant 1.

**Exercise 4.1.** Find a cell decomposition for a Grassmann manifold $\frac{U(4)}{U(2)\times U(2)}$ of all 2-dimensional complex subspaces in $\mathbb{C}^4$ such that all cells are even-dimensional.

**Exercise 4.2.** Prove that a standard embedding $\mathbb{C}P^{n-k} \hookrightarrow \mathbb{C}P^n$ induces a surjective map on cohomology.

**Exercise 4.3.** Prove that $SU(2)$ acts on a unit sphere $S^3 \subset \mathbb{C}^2$ freely and transitively.

**Remark 4.1.** This implies that $SU(2) = S^3$.

**Exercise 4.4.** Prove that $SU(n)$ is fibered over $S^{2n-1}$ with fiber $SU(n-1)$.

**Exercise 4.5.** Let $U(\mathbb{H}, 1)$ be a group of unitary quaternions. Prove that $U(\mathbb{H}, 1)$ is isomorphic to $SU(2)$.

**Exercise 4.6.** Construct a double cover $SU(2) \longrightarrow SO(3)$.

**Exercise 4.7.** Construct a double cover $SU(2) \times SU(2) \longrightarrow SO(4)$.