Research overview

Misha Verbitsky, December 2011[1]

1 Hodge theory on hyperkähler manifolds and its applications

In [V90], [V94], [V95:1], [V95:2], I studied the applications of Hodge theory for topology of hyperkähler manifolds. It was shown that cohomology of a hyperkähler manifold admit an action of the Lie group $Sp(1,1)$, which is similar to Lefschetz' $SL(2)$-action. This was used to compute the cohomology algebra of a hyperkähler manifold, showing that its part generated by $H^2(M)$ is symmetric, up to the middle dimension. Among applications of these results, a proof of Mirror Conjecture for hyperkähler manifolds, and better understanding of hyperkähler subvarieties of hyperkähler manifolds and coherent sheaves.

The latest achievement in this direction was a proof of global Torelli theorem, [V09:2], which was a subject of Bourbaki seminar talk by Dan Huybrechts ([H]); this result already has many applications ([AV10], [A], [BS], [Mn], most notably – a proof of Kawamata-Morrison's cone conjecture for hyperkähler manifolds by Eyal Markman ([Ma]).

2 Trianalytic subvarieties of hyperkähler manifolds

The papers [V93], [V94], [V96:1], [V96:2], [V97:1], [V97:2], [KV98:1], [V98], [KV98:2] and [V03:4] deal with trianalytic subvarieties in hyperkähler manifolds. These are subvarieties which are complex analytic with respect to three complex structures I, J, K. It was shown that all complex subvarieties of a generic hyperkähler manifold are trianalytic. Also all deformations of trianalytic subvarieties are again trianalytic, and their deformation space is singular hyperkähler.

I have studied singularities of singular hyperkähler varieties, and shown that a normalization of such variety is smooth and hyperkähler. This applies to trianalytic subvarieties, which are examples of singular hyperkähler spaces.

These results were applied to Hilbert schemes of points on K3 and generalized Kummer varieties. For Hilbert schemes of points on K3, it was shown that its generic deformation has no subvarieties. For generalized Kummer varieties, a similar attempt (joint with D. Kaledin) was foiled by

[1] `verbit@mccme.ru`

our imperfect understanding of birational geometry of holomorphic symplectic manifolds. Soon after publishing this paper, we found a counterexample to one of our statements.

The birational geometry of holomorphic symplectic manifolds was studied by D. Kaledin in several papers in much detail. Our joint work in this direction resulted in [KV00], where a deformation theorem, analoguous to Bogomolov-Tian-Todorov, was obtained for non-compact holomorphic symplectic manifolds. Also, in [V99] I have studied quotient singularities of holomorphic symplectic manifolds admitting a holomorphic symplectic resolution, and proved that such singularities are always quotients by groups generated by symplectic reflections.

3 Holomorphic Lagrangian subvarieties

In a joint paper with Geo Grantcharov [GV10], we put the results on trianalytic subvarieties into a more general framework of calibrated geometry. We considered calibrations on hyperkähler manifolds which can be expressed as polynomials of holomorphic symplectic forms. One of these calibrations calibrates precisely the trianalytic subvarieties. We found another such calibration, calibrating holomorphic Lagrangian subvarieties. Surprisingly, for the latter calibration to be closed, a manifold needs not to be hyperkähler: we have shown that such a calibration exists on a hypercomplex manifold, provided that its holonomy lies in $SL(n, \mathbb{H})$.

The holomorphic Lagrangian subvarieties appear in another context connected to the Mirror Symmetry. A Strominger-Yau-Zaslow (SYZ) conjecture postulates existence of special Lagrangian fibrations pm Calabi-Yaus; the duality of the fibers of these fibrations is responsible for Mirror Symmetry.

Any holomorphic Lagrangian subvariety on a hyperkaehler manifold is speacial Lagrangian with respect to some other complex structure, hence existence of holomorphic Lagrangian fibrations can be understand as a special case of the SYZ conjecture. I wrote a couple of papers ([V08:4], [V09:1]) dealing with this conjecture, and proved, in particular, that a hyperkaehler manifold which has no cohomological obstructions to existence of holomorphic Lagrangian fibrations admits, at least, a coisotropic subvariety.

4 Coherent sheaves on hyperkähler manifolds

The papers [V92], [KV96], [V97:3], [V01:1], [MV06] are about hyperholomorphic bundles on hyperkähler manifolds. These are bundles with Hermitian connection with curvature of type (1,1) with respect to all complex structures induced by the hyperkähler structure.

It was shown that all stable bundles on generic hyperkähler manifolds admit a hyperholomorphic connection, which is unique. Conversely, every hyperholomorphic bundle is a direct sum of stable bundles.

The moduli spaces of such bundles wre shown to be singular hyperkähler, and the deformation unobstructed (except the first obstruction, known as Yoneda product).

Later, this notion was extended to coherent sheaves. Using results about trianalytic subvarieties, I have shown that a deformation of a hyperholomorphic bundle over a generic hyperkähler manifold M remains non-singular, unless M contains trianalytic subvarieties of complex codimension 2 (in the cases of a Hilbert scheme of K3 and the generalized Kummer variety, all trianalytic subvarieties have codimension > 2, except for 4-dimensional generalized Kummer).

If the deformation spaces of hyperholomorphic bundles are positive-dimensional (which is unknown yet), this should lead to new examples of hyperkähler manifolds.

In [KV96], a non-Hermitian version of this notion was studied. We have shown that if the Hermitian assumption is dropped, the non-Hermitian hyperholomorphic connection on M becomes essentially the same as a holomorphic structure on the lifting of the corresponding bundle to the twistor space. In [V02], [V03:3] this approach was used to study the category of coherent sheaves on generic K3 surfaces and tori. It was shown that this category is independent from the choice of a generic K3 or a torus of a given dimension.

5 Moduli spaces of framed instanton bundles on CP^3 and the rational curves on the twistor space

In [KV96], we constructed a correspondence between stable vector bundles on a twistor space of a hyperkaähler manifopld and rational curves in a twistor space of another hyperkähler manifold. This observation was used in a recent work with Marcus Jardim ([JV10], [JV11]).

We have shown that the moduli space M of holomorphic vector bundles on CP^3 that are trivial along a line is isomorphic (as a complex manifold) to a subvariety in the moduli of rational curves of the twistor space of the moduli space of framed instantons on $\mathbb{C}^2$, called the space of twistor sections. This space admits an interesting geometric structure, called **a trisymplectic structure**.

A trisymplectic structure on a complex $2n$-manifold is a triple of holomorphic symplectic forms such that any linear combination of these forms has rank $2n$, n or 0. We have shown that a trisymplectic manifold is equipped with a holomorphic 3-web and the Chern connection of this 3-web is holomorphic, torsion-free, and preserves the three symplectic forms. Then we constructed a trisymplectic structure on the moduli of regular rational curves in the twistor space of a hyperkaehler manifold, and defined a trisymplectic reduction of a trisymplectic manifold, which is a complexified form of a hyperkaehler reduction. We proved that the trisymplectic reduction in the space of regular rational curves on the twistor space of a hyperkaehler manifold M is compatible with the hyperkaehler reduction on M.

As an application of these geometric ideas, we considered the ADHM construction of instantons. We have shown that the moduli space of rank r, charge c framed instanton bundles on CP^3 is a smooth, connected, trisymplectic manifold of complex dimension $4rc$. In particular, it follows that the moduli space of rank 2, charge c instanton bundles on CP^3 is a smooth complex manifold dimension $8c-3$, thus settling a 30-year old conjecture of Barth and Hartshorne.

6 Hodge theory on hypercomplex manifolds and HKT-geometry

A **hypercomplex manifold** is a manifold with three complex structures I, J, K satisfying quaternionic relations. It is called **quaternionic Hermitian** if it has a quaternionic-invariant Riemannian structure. With each quaternionic Hermitian manifold (M, I, J, K, g), one can associate its canonical $(2,0)$-form $\Omega = \omega_J + \sqrt{-1}\omega_K$. If this form is closed, M is called hyperkähler (this is one of possible definitions). If $\partial\Omega = 0$, (M, I, J, K, g) is called **an HKT-manifold** ("hyperkähler with torsion").

An HKT form is in many ways similar to a Kähler structure. One can define a potential, a version of Hodge theory, Kähler class and so on. In papers [V01:2], [V03:2], [V04:2], [V04:3], [V06:1], [BDV07] I studied hypercomplex geometry from this point of view. The Hodge theory (including Lefschetz-type $SL(2)$-action) was constructed for HKT manifolds with trivial canonical bundle. As an application, it was shown that a compact hypercomplex manifold which admits a Kähler metric also admits a hyperkähler structure. In another application (jointly with I. Dotti and M. L. Barbieris) it was shown that a hypercomplex nilmanifold admits an HKT structure if and only if it is abelian.

This result was also useful in hyperkähler geometry, where a strong vanishing result was shown, based on this Lefschetz -type $SL(2)$-action. It was shown that cohomology of a holomorphic line bundle L with $-c_1(L)$ outside of a dual Kähler cone vanish after the middle dimension. Moreover, form any holomorphic bunldle M, $H^i(B \otimes L^N) = 0$, for N sufficiently big, and $i > \frac{1}{2}\dim_{\mathbb{C}} M$.

The version of Hodge theory developed for the study of HKT manifolds, was also useful in other geometric sutuations, namely, for G_2-manifolds and nearly Kähler manifolds ([V05:1], [V05:2], [V05:3]). For nearly Kähler manifolds, the Hodge relations were sufficient to obtain the Hodge decomposition. During the work on Hodge theory of G_2-manifolds, many concepts of complex algebraic geometry were adapted to work on G_2-manifolds. This way I obtained some basic results in the theory of calibrated plurisubharmonic functions, later rediscovered by Harvey and Lawson in a different (and more systematic) framework.

This theory was used to study coherent sheaves on hyperkähler manifolds, and (jointly with Semyon Alesker) Calabi-Yau problem in HKT geometry.

7 Complex structures and special holonomies

Since early 2000-ies, I was trying to apply complex (more precisely, Hodge-theoretic) methods to manifolds with special holonomy (G_2-manifolds and their locally conformal analogues, obtained as Riemannian cones over nearly Kähler manifolds). This approach brought some interesting results. In particular, I was able to construct an analogue of Hodge theory and Kähler relations for manifolds with special holonomy ([V01:2], [V05:1], [V05:2], [V05:3]).

However, one of the main applications of the Kähler formalism still missing: nobody seems to know how to prove formality for G_2-manifolds. It seems that some complex structure is still required, e.g. in the form of a twistor space.

During the course of this work, I realized that a knot space of a G_2-manifold has a formally integrable Kähler structure; also, a CR-holomorphic twistor space was constructed ([V10:1], [V10:2]). This structure is analogous to the Brylinski's formally integrable Kähler structure on a know space of a Riemannian 3-manifold, but in the G_2-case the formal integrability of this Kähler structure depends on differential-geometric properties of a manifolds (and, in fact, it is equivalent to the holonomy condition).

It seems that these structures should play in G_2-geometry the same role as the usual twistor spaces play in quaternionic geometry.

8 Plurisubharmonic functions in hypercomplex geometry

In [AV05], [AV08] we studied the plurisubharmonic function on a hypercomplex manifold M. If M is $\mathbb{H}^n$, these are functions which are subharmonic on all 1-dimensional quaternionic planes. The theory of quaternionic plurisubharmonic functions is deeply related with HKT-geometry, because HKT-potentials are precisely the C^2-functions which are strictly plurisubharmonic. We formulated a version of Calabi conjecture for HKT manifolds and proved uniqueness of its solution and C^0-estimates. In [V08:3], it was shown that an HKT metric is Calabi-Yau HKT if and only if it is balanced (satisfies $d\omega^{\dim_{\mathbb{C}} M-1} = 0$).

The appropriate notion of positivity (called K-positivity there) originates in [V01:1], where it was used to study direct image of hyperholomorphic sheaves. In order to prove stability, an L^2-estimation of singularities was required. It was obtained by a clumsy approximation argument. In [V08:1] and [V08:2] the theory of calibrated plurisubharmonic function was developed, in parallel with the usual complex analysis, and the stability of higher direct images of hyperholomorphic sheaves was obtained in a clean way as an application of this theory.

Another application of this formalism was obtained in a joint paper with Grantcharov, [GV10], where new calibrations on hyperkähler and HKT-manifolds were constructed.

9 Locally conformally Kähler geometry

A **locally conformally Kähler manifold** (LCK-manifold) is a complex manifold which is covered by a Kähler, with the deck transform acting by holomorphic homotheties. An impotrant special case is so-called **Vaisman manifolds**, which are covered by a Kähler manifold where $\mathbb{R}$ acts by holomorphic homotheties. Similarly one defines a locally conformally hyperkähler manifold. In [V03:1], I obtained a structure theorem for locally conformally hyperkähler manifolds, reducing their classification to classification of 3-Sasakian manifolds, which is due to Boyer, Galicki, Demailly and Kollar.

Since then, I collaborated with Liviu Ornea in a series of papers ([OV03:1], [OV03:12], [OV04], [OV06:1], [OV06:2], [OV06:3], [OV07:1], [OV09:1], [OV09:2], [OV10:1]) on locally conformally Kähler geometry.

We have established a structure theorem for Vaisman manifolds, reducing the Vaisman geometry to Sasakian geometry, and proved that a Vaisman manifold admits a holomorphic immersion in a linear Hopf manifold. This was used to obtain similar immersion results for Sasakian manifold, proving that they always admit a CR-holomorphic embedding to a contact sphere. These results were also used to characterize CR-manifolds admitting Sasakian structure in terms of their automorphism group.

In attempt to understand the immersion theorem, we invented a new class of LCK-manifold, called **LCK-manifolds with automorphic potential**. This is an intermediate class between the Vaisman manifolds and LCK-manifolds. Unlike the Vaisman and LCK-manifolds, LCK-manifolds with automorphic potential are stable under small complex deformations. Also, such manifolds admit holomorphic embedding to linear Hopf manifold. This result can be understood as a locally conformally Kähler version of a Kodaira embedding theorem.

Later on, we found that LCK-manifolds with automorphic potential can be characterized in terms of vanishing of a certain cohomology class, which can be understood as a holomorphic version of Morse-Novikov cohomology. This was used to characterize such manifolds in terms of their automorphism groups, and to obtain important results about topology, describing their topology in terms of topology of certain algebraic varieties.

10 Toric and elliptic fibrations

In [V04:1], I studied vector bundles and coherent sheaves on compact complex non-Kähler manifolds admitting a principal elliptic fibration with a Kähler base. There are many examples of such manifolds coming from physics, hypercomplex geometry and LCK-geometry. Positive elliptic fibrations were defined (a big class including quasi-regular Vaisman and Calabi-Eckmann manifolds). I have shown that any stable sheaf on a positive elliptic fibration of $\dim_{\mathbb{C}} \geq 3$ is lifted from a base (up to a product with line bundle). This implies, in particular, that all coherent sheaves are filtrable (in constract to the case of non-Kähler surfaces, where coherent sheaves are rarely filtrable). In [V04:4], filtrability was generalized to Hopf manifolds of $\dim_{\mathbb{C}} \geq 3$. In [V07:1], the notion of positive toric fibration was discussed, including the positive elliptic fibrations and invariant complex structures on compact Lie groups. It was shown that all connected subvarieties in a positive toric fibration are contained in a fiber, or lifted from a base.

In [OV10:2], this approach was used to show that the Oeljeklaus-Toma manifolds (multi-dimensional generalizations of Inoue surfaces) have no non-trivial complex subvarieties.

References

[A] Apostol Apostolov, *Moduli spaces of polarized irreducible symplectic manifolds are not necessarily connected*, arXiv:1109.0175.

[BS] Samuel Boissiere, Alessandra Sarti, *A note on automorphisms and birational transformations of holomorphic symplectic manifolds*, arXiv:0905.4370.

[H] Daniel Huybrechts, *A global Torelli theorem for hyperkaehler manifolds (after Verbitsky)*, arXiv:1106.5573.

[Ma] Markman, E. *A survey of Torelli and monodromy results for holomorphic-symplectic varieties*, Proceedings of the conference "Complex and Differential Geometry", Springer Proceedings in Mathematics, 2011, Volume 8, 257–322, arXiv:math/0601304.

[Mn] Giovanni Mongardi, *Symplectic involutions on deformations of* $K3^{[2]}$, arXiv:1107.2854.

[V90] Verbitsky M. *On the action of a Lie algebra SO(5) on the cohomology of a hyperkähler manifold,* Func. Analysis and Appl. 24(2) p 70-71 (1990).

[V92] Verbitsky M., *Hyperholomorphic bundles over the hyperkähler manifolds,* alg-geom 9307008, Journ. of Alg. Geom., **5** no. 4 (1996) pp. 633-669.

[V93] Verbitsky M., *Hyperkähler and holomorphic symplectic geometry I.* alg-geom 9307009, also in Journ. of Alg. Geom., vol. 5 no. 3 pp. 401-415 (1996).

[V94] Verbitsky, M., *Hyperkähler embeddings and holomorphic symplectic geometry II.* alg-geom 9403006, 14 pages, also in GAFA vol. 5 no. 1 (1995) pp. 92-104

[V95:1] Verbitsky M., *Cohomology of compact hyperkähler manifolds and its applications* alg-geom 9511009, also in: GAFA vol. 6 no. 4 pp. 601–612 (1996)

[V95:2] Verbitsky M., *Mirror Symmetry for hyperkähler manifolds,* alg-geom 9512195 (published in Mirror Symmetry III, International Press).

[KV96] Kaledin, D., Verbitsky, M., *Non-Hermitian Yang-Mills connections,* alg-geom 9606019, also in Selecta Math. 4 (1998) 279-320

[V96:1] Verbitsky M., *Deformations of trianalytic subvarieties of hyperkähler manifolds*, alg-geom 9610010 (Selecta Math., New ser. **4** (1998) 447-490)

[V96:2] Verbitsky M., *Desingularization of singular hyperkähler varieties I,* alg-geom 9611015, also Math. Res. Lett. **4** (1997), no. 2-3, pp. 259–271.

[V97:1] Verbitsky M., *Hypercomplex Varieties*, alg-geom 9703016 (Comm. Anal. Geom. 7 (1999), no. 2, 355–396.)

[V97:2] Verbitsky M., *Trianalytic subvarieties of the Hilbert scheme of points on a K3 surface,* alg-geom 9705004, GAFA, Vol. 8 (1998) 732-782.

[V97:3] Verbitsky M., *Hyperholomorphic sheaves and new examples of hyperkähler manifolds,* alg-geom 9712012 (published as a part of a book "Hyperkähler manifolds", by M. Verbitsky and D. Kaledin, Mathematical Physics, 12, International Press, 1999.)

[KV98:1] Kaledin D., Verbitsky M., *Trianalytic subvarieties of generalized Kummer varieties*, math.AG 9801038, published in Int. Math. Res. Not. 1998, No. 9, 439–461.

[V98] Verbitsky M., *Wirtinger numbers and holomorphic symplectic immersions*, math.AG:9812077, Selecta Math. (N.S.) 10 (2004), no. 4, 551–559.

[KV98:2] Kaledin D., Verbitsky M., *Partial resolutions of Hilbert type, Dynkin diagrams, and generalized Kummer varieties*,
math.AG 9812078 (33 pages).

[V99] M. Verbitsky, *Holomorphic symplectic geometry and orbifold singularities*, math.AG 9903175 (17 pages, LaTeX 2e), published in Asian Journ. of Math., Vol. 4 (2000), No. 3, pp. 553-564

[KV00] D. Kaledin, M. Verbitsky, *Period map for non-compact holomorphically symplectic manifolds*, math.AG:0005007, GAFA 12 (2002), no. 6, 1265–1295.

[V01:1] M. Verbitsky, *Hyperholomorpic connections on coherent sheaves and stability*, 43 pages, math.AG:0107182

[V01:2] M. Verbitsky, *Hyperkähler manifolds with torsion, supersymmetry and Hodge theory*, math.AG:0112215, Asian J. of Math., Vol. 6, No. 4, pp. 679-712 (2002)

[V02] M. Verbitsky, *Coherent sheaves on general K3 surfaces and tori*, math.AG:0205210, 63 pages, Pure Appl. Math. Q. 4 (2008), no. 3, part 2, 651–714.

[V03:1] M. Verbitsky, *Vanishing theorems for locally conformal hyperkähler manifolds*, math.DG:0302219, 41 pages, Proc. of Steklov Institute, vol. 246, 2004, pp. 54-79.

[V03:2] M. Verbitsky, *Hyperkähler manifolds with torsion obtained from hyperholomorphic bundles* math.DG:0303129, Math. Res. Lett. 10 (2003), no. 4, 501–513.

[V03:3] Verbitsky, M., *Coherent sheaves on generic compact tori*, math.AG:0310329, CRM Proc. and Lecture Notices vol. 38 (2004), 229-249

[V03:4] Verbitsky, M., *Subvarieties in non-compact hyperkähler manifolds*, math.AG:0312520, Math. Res. Lett. vol. 11 (2004), no. 4, pp. 413-418

[OV03:1] L. Ornea, M. Verbitsky, *Structure theorem for compact Vaisman manifolds*, math.DG:0305259, Math. Res. Lett, 10(2003), no. 5-6, 799-805

[OV03:12] L. Ornea, M. Verbitsky, *Immersion theorem for Vaisman manifolds*, math.AG:0306077, 28 pages, Math. Ann. 332 (2005), no. 1, 121–143.

[V04:1] Verbitsky, M., *Stable bundles on positive principal elliptic fibrations*, math.AG:0403430, 17 pages, Math. Res. Lett. 12 (2005), no. 2-3, 251–264.

[V04:2] Verbitsky, M., *Hypercomplex structures on Kähler manifolds*, math.AG:0406390, 10 pages, GAFA 15 (2005), no. 6, pp. 1275-1283.

[V04:3] Verbitsky, M., *Hypercomplex manifolds with trivial canonical bundle and their holonomy*, math.DG:0406537, 13 pages, "Moscow Seminar on Mathematical Physics II", Translations of AMS, vol. 221 (2007).

[OV04] L. Ornea, M. Verbitsky, *Locally conformal Kähler manifolds with potential*, math.AG:0407231, Mathematische Annalen, Vol. 248 (1), 2010, pp. 25-33.

[V04:4] Verbitsky, M., *Holomorphic bundles on diagonal Hopf manifolds*, 20 pages, math.AG:0408391, Izvestiya Math., 2006, **70**, no. 5, pp. 13-31.

[V05:1] Verbitsky, M., *Manifolds with parallel differential forms and Kähler identities for G_2-manifolds*, math.DG:0502540, Journal of Geometry and Physics, vol. 61 (6), pp. 1001-1016 (2011).

[V05:2] Verbitsky, M., *An intrinsic volume functional on almost complex 6-manifolds and nearly Kähler geometry,* arXiv:math:0507179, Pacific J. of Math., Vol. 235, No. 2 (2008), 323-344

[AV05] Semyon Alesker, Misha Verbitsky, *Plurisubharmonic functions on hypercomplex manifolds and HKT-geometry*, 34 pages, Journal of Geometric Analysis, **16**, 3, Sep. 2006, pp. 375-400

[V05:3] Verbitsky, M., *Hodge theory on nearly Kähler manifolds*, Geometry and Topology, 2011, No. 15. pp. 2111-2133.

[V06:1] Verbitsky, M., *Quaternionic Dolbeault complex and vanishing theorems on hyperkahler manifolds*, math.AG:0604303, 30 pages, Compos. Math. 143 (2007), no. 6, 1576–1592.

[OV06:1] L. Ornea, M. Verbitsky, *Sasakian structures on CR-manifolds*, Geom. Dedicata 125 (2007), 159–173.

[OV06:2] Liviu Ornea, Misha Verbitsky, *Einstein-Weyl structures on complex manifolds and conformal version of Monge-Ampere equation*, Bull. Math. Soc. Sci. Math. Roumanie (N.S.) 51(99) (2008), no. 4, 339–353.

[OV06:3] Liviu Ornea, Misha Verbitsky, *Embeddings of compact Sasakian manifolds*, math.DG:0609617, 10 pages, Math. Res. Lett. 14 (2007), no. 4, 703–710.

[MV06] Ruxandra Moraru, Misha Verbitsky, *Stable bundles on hypercomplex surfaces*, Cent. Eur. J. Math. 8 (2010), no. 2, 327–337.

[V07:1] Misha Verbitsky, *Positive toric fibrations*, J. Lond. Math. Soc. (2) 79 (2009), no. 2, 294–308.

[OV07:1] Liviu Ornea, Misha Verbitsky, *Morse-Novikov cohomology of locally conformally Kähler manifolds*, arXiv:0712.0107, J. Geom. Phys. 59 (2009), no. 3, 295–305.

[BDV07] Maria Laura Barberis, Isabel G. Dotti, Misha Verbitsky, *Canonical bundles of complex nilmanifolds, with applications to hypercomplex geometry*, arXiv:0712.3863, Math. Res. Lett. 16 (2009), no. 2, 331–347.

[V08:1] Misha Verbitsky, *Plurisubharmonic functions in calibrated geometry and q-convexity*, arXiv:0712.4036, Math. Z., Vol. 264, No. 4, pp. 939-957 (2010).

[V08:2] Misha Verbitsky, *Positive forms on hyperkahler manifolds*, Osaka J. Math. Volume 47, Number 2 (2010), 353-384.

[AV08] Semyon Alesker, Misha Verbitsky, *Quaternionic Monge-Ampere equation and Calabi problem for HKT-manifolds*, arXiv:0802.4202, Israel Journal of Mathematics, Vol. 176, Number 1 (2010), 109-138.

[V08:3] Misha Verbitsky, *Balanced HKT metrics and strong HKT metrics on hypercomplex manifolds,* arXiv:0808.3218, Math. Res. Lett. 16 (2009), no. 4, 735–752.

[V08:4] Misha Verbitsky, *Hyperkahler SYZ conjecture and semipositive line bundles*, arXiv:0811.0639, GAFA 19, No. 5, 1481-1493 (2010).

[OV09:1] Liviu Ornea, Misha Verbitsky, *Topology of locally conformally Kahler manifolds with potential*, arXiv:0904.3362, Int. Math. Res. Not. 2010, No. 4, 717-726 (2010).

[OV09:2] Liviu Ornea, Misha Verbitsky, *Automorphisms of locally conformally Kahler manifolds*, arXiv:0906.2836, 9 pages

[V09:1] M. Verbitsky, *Parabolic nef currents on hyperkaehler manifolds*, arXiv:0907.4217.

[V09:2] M. Verbitsky, *A global Torelli theorem for hyperkahler manifolds*, arXiv:0908.4121.

[GV10] Gueo Grantcharov, Misha Verbitsky, *Calibrations in hyperkahler geometry*, arXiv:1009.1178

[OV10:1] Liviu Ornea, Misha Verbitsky *Locally conformally Kahler manifolds admitting a holomorphic conformal flow*, arXiv:1004.4645.

[OV10:2] Ornea L., Verbitsky, M., *Oeljeklaus-Toma manifolds admitting no complex subvarieties*, Mathematical Research Letters, 2011, No. 04 (18). pp. 747-754.

[V10:1] M. Verbitsky, *A CR twistor space of a G2-manifold*, Differential Geometry and its Applications, 2011, No. 29 (apr), pp. 101-107

[V10:2] M. Verbitsky, *A formally Kahler structure on a knot space of a G2-manifold*, arXiv:1003.3174, accepted by Selecta Math.

[AV10] Sasha Anan'in, Misha Verbitsky, *Any component of moduli of polarized hyperkaehler manifolds is dense in its deformation space*, arXiv:1008.2480.

[JV10] Jardim M., Verbitsky, M. *Moduli spaces of framed instanton bundles on CP^3 and twistor sections of moduli spaces of instantons on C^2*, Adv. Math., 2011, No. 227. pp. 1526-1538.

[JV11] Jardim M., Verbitsky, M. *Trihyperkahler reduction and instanton bundles on CP^3*, arXiv:1103.4431.