Test assignment #2

Rules: Please solve this in class, before 21:30 February 04, 2013, and give me the written solutions. The score for the test is computed using the formula $s = 3p - (\max(p-4,0))$. Results will be announced at `http://bogomolov-lab.ru/KURSY/GEOM-2013/`

Exercise 1.1. Consider **the Moebius strip** M as a quotient space of $\mathbb{R}\times[0,1]$ with opposite lines glued together with reverse orientation. Construct a closed embedding of M to $\mathbb{R}^4$.

Exercise 1.2 (4 points). Construct a closed embedding of Moebius strip to $\mathbb{R}^3$, or prove that it does not exist.

Exercise 1.3. Construct an embedding of $(S^1)^3$ to $\mathbb{R}^4$, or prove that it does not exist.

Exercise 1.4. Construct an embedding of $S^1 \times S^2$ to $\mathbb{R}^4$, or prove that it does not exist.

Exercise 1.5. Let M be an n-dimensional manifold. Construct a smooth, surjective map from M to the torus $(S^1)^n$.

Exercise 1.6. Let R be a ring of continuous $\mathbb{R}$-valued functions on a topological space M, and $I \subset R$ a prime ideal. Prove that $I^2 = I$.

Exercise 1.7 (3 points). Let R be a ring of continuous $\mathbb{R}$-valued functions on a compact topological space M, and $I \subset R$ an ideal. Prove that there exists $Z \subset M$ such that I is an ideal of all functions vanishing at Z, or find a counterexample.

Exercise 1.8 (3 points). Let R be a ring of germs of continuous $\mathbb{R}$-valued functions at a point $x \in M$ of a manifold M, and $I \subset R$ a prime ideal. Prove that I is maximal, or find a counterexample.

Definition 1.1. A sheaf $\mathcal{B}$ is called **flasque** if any restriction map $\mathcal{B}(U) \longrightarrow \mathcal{B}(V)$ is surjective.

Exercise 1.9. Prove that any flasque sheaf is soft.

Exercise 1.10. For a given sheaf $\mathcal{B}$, find a sheaf monomorphism $\mathcal{B} \hookrightarrow \mathcal{B}'$ to a flasque sheaf.

Definition 1.2. A sheaf $\mathcal{I}$ is called **injective** if for any sheaf morphism $\mathcal{B} \stackrel{\phi}{\longrightarrow} \mathcal{I}$ and a monomorphism $\mathcal{B} \hookrightarrow \mathcal{B}'$, the map ϕ can be extended to a morphism $\mathcal{B}' \stackrel{\phi}{\longrightarrow} \mathcal{I}$.

Exercise 1.11. Prove that any injective sheaf is flasque.