LCK manifolds 2: LCK manifolds and Hermitian structures

Exercise 2.1. Let θ a closed 1-form on M $d_\theta = d - \theta$, and $H^*_\theta(M)$ cohomology of the complex $\Lambda^*(M), d_\theta$ ("Morse-Novikov cohomology"). Prove that $H^i_\theta(M) = H^i(M)$ when θ is exact.

Exercise 2.2. Let (M, ω, θ) be a compact LCK manifold, satisfying $d^c\theta = 0$. Prove that $\theta = 0$.

Exercise 2.3. Let (M, ω, θ) be a compact LCK manifold, satisfying $dd^c\omega = 0$, and $\dim_{\mathbb{C}} M > 2$. Prove that $\theta = 0$.

Definition 2.1. Let M be a complex manifold, $\dim_{\mathbb{C}} M = n$. A Hermitian metric on M is called **balanced** if $d\omega^{n-1} = 0$.

Exercise 2.4. Prove that a classical Hopf manifold $C^n/x \sim \lambda x$ does not admit a balanced metrics.

Exercise 2.5. Let ω be a non-degenerate 2-form on a $2n$-dimensional smooth manifold, and $d(\omega^k) = 0$ for some k satisfying $0 < k < n - 1$. Prove that $d\omega = 0$.

Exercise 2.6. Let (M, ω, θ) be a compact LCK manifold, $\dim_{\mathbb{C}} M > 2$, $dd^c\omega = 0$. Prove that $\theta = 0$.