4 Exercises for "Kähler geometry", lecture 4

University of Science and Technology, China, Hefei

Problem 4.1: Suppose that $\dim_{\mathbb{R}} M = 2$. Prove that any almost complex structure on M is integrable.

Problem 4.2: Construct a non-integrable almost complex structure on a manifold M with $\dim_{\mathbb{R}} M = 4$.

Problem 4.3: Let (M, I) be a smooth almost complex manifold equipped with a transitive action of a group G. Assume that I is G-invariant (such a manifold is called **homogeneous**). Assume, moreover, that for some $x \in M$ there exists an element $\tau_x \in G$ fixing x. Consider the induced action of τ_x on $T_x M$; denote this operator by τ.

(a) Suppose that $\tau = \lambda \operatorname{Id}$, where $\lambda \in \mathbb{R}$. Prove that for all $\lambda \neq 1$, the almost complex structure I is integrable.

(b) Construct examples of such (M, I), G and τ_x for each $\lambda \in \mathbb{R}$.

(c) Construct a homogeneous almost complex manifold which is not integrable.

(d) Suppose that τ is not a scalar, but all its eigenvalues α_i satisfy $9 < |\alpha_i| < 10$. Prove that the almost complex structure I is integrable.

Please bring these assignments in writing to the next lecture (Monday, 16.07.2012).