Hyperkähler manifolds and reflection lattices

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K3 surfaces

DEFINITION: A K3 surface is a complex surface (that is, a compact complex 2-manifold M) which satisfies $\pi_1(M) = 0$ and has trivial canonical bundle.

EXAMPLE: All smooth quartics in $\mathbb{C}P^3$ are K3.

CLAIM: The Hodge diamond of a K3 surface:

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array}$$

In particular, $\dim H^{2,0}(M)$ (this group is generated by the holomorphic section of the canonical bundle) and $\dim H^{1,1}(M) = 20$.

Projective K3 surfaces

DEFINITION: The Neron-Severi lattice of a K3 is $H^{1,1}(M,\mathbb{Z}) := H^{1,1}(M) \cap H^2(M,\mathbb{Z})$.

REMARK: The intersection form on $H^{1,1}(M)$ has signature (1,19) (Hodge index theorem). Therefore, **one of the three alternatives occurs:**

(a) the signature of the intersection form on $H^{1,1}(M,\mathbb{Z})$ is $(1,k)$,

(b) this form is degenerate,

(c) It is negative definite.

In the case (a), K3 is projective:

THEOREM: A K3 is projective if and only if the there exists $\eta \in H^{1,1}(M,\mathbb{Z})$ such that $\int_M \eta \wedge \eta > 0$.

The Kähler cone

THEOREM: (Y.-T. Siu)
A K3 surface **is always Kähler.**

DEFINITION: The Kähler cone $\operatorname{Kah}(M) \subset H^{1,1}(M)$ of a Kähler manifold is the set of all cohomology classes of all Kähler forms on M.
REMARK: The Kähler cone **is a convex, open cone in $H^{1,1}(M,\mathbb{R})$.**

DEFINITION: The **positive cone** $\operatorname{Pos}(M)$ is one of the connected component of the set $\{\eta \in H^{1,1}(M,\mathbb{R}) \ \ | \ \ \int_M \eta \wedge \eta > 0\}$ which contains $\operatorname{Kah}(M)$.

The Kähler cone and (-2)-classes

DEFINITION: A **(-2)-class** on a K3 surface is $\eta \in H^{1,1}(M,\mathbb{Z})$ such that $\int_M \eta\wedge\eta = -2$.

REMARK: From Riemann-Roch formula it follows that **any (-2)-class η, either η or $-\eta$ is effective,** that is, equal to the fundamental class of a curve (we can also show that this curve is a union of smooth rational curves and elliptic curves, smooth or nodal).

REMARK: This implies that **a class in the orthogonal complement $\eta^\perp$ of a (-2)-class cannot be Kähler.**

This observation (together with the Demailly-Paun characterization of Kählerness) lies in the foundation of the following

THEOREM: Let M be a K3 surface, and $\mathfrak{S} \subset H^{1,1}(M,\mathbb{Z})$ the set of all (-2)-classes. Consider the decomposition of onto the union of connected components. Then **the Kähler cone of M is one of the connected components of $\operatorname{Pos}(M)\backslash \bigcup_{s\in\mathfrak{S}} s^\perp$.**

DEFINITION: The orthogonal complements $s^\perp, s\in\mathfrak{S}$ are called **the walls** of the Kähler cone.

Automorphisms of K3 surfaces

The following theorem immediately follows from the Torelli theorem.

THEOREM: Let M be a K3 surface. The group $\operatorname{Aut}(M)$ of holomorphic isometries of M **is naturally identified with the subgroup of $O(H^2(M,\mathbb{Z}))$ which preserves the Kähler cone.**

REMARK: The holomorphically symplectic form of M is any non-zero element of $H^{2,0}(M)$. Therefore, **an automorphism of M is holomorphically symplectic if and only if it acts trivially on $H^{2,0}(M)$.** In particular, **the group of holomorphically symplectic automorphisms of a K3 is the kernel of the natural map $\operatorname{Aut}(M) \to \operatorname{Aut}(H^{2,0}(M)) = \mathbb{C}^*$.**

Symplectic automorphisms of K3 surfaces

DEFINITION: A group is called **vitrually free abelian** if it contains a free abelian subgroup of finite index.

Proposition 1: (Oguiso)
Let M be a K3 surface, and $H^{1,1}(M,\mathbb{Z})$ its Neron-Severi lattice. Then the following alternatives occur.

(a) If the intersection form on $H^{1,1}(M,\mathbb{Z})$ is negative definite, then **the kernel of** $\operatorname{Aut}(M) \stackrel{\Psi}{\to} \operatorname{Aut}(H^{2,0}(M)) = \mathbb{C}^*$ **is finite, and the image is virtually free abelian.**

(b) If the intersection form on $H^{1,1}(M,\mathbb{Z})$ is degenerate, **the image and the kernel of the map Ψ are virtually free abelian.**

(c) If M is projective, **the image of Ψ is finite.**

Proof: Let $\mathbb{T}$ be the transcendental Hodge lattice of M, that is, the smallest rational Hodge substructure containing $H^{2,0}(M)$). Then $\ker\Psi$ faithfully acts on $\mathbb{T}$ by isometries. If M is projective, $\ker\Psi$ is compact because the $\mathbb{T}$ is polarized, hence $\operatorname{Aut}(\mathbb{T})$ is finite. If $H^{1,1}(M,\mathbb{Z})$ is negative definite, its group of isometries is finite, hence $\ker\Psi$ is finite. If the intersection form on $H^{1,1}(M,\mathbb{Z})$ is degenerate, the groups of Hodge isometries of $\mathbb{T}$ and of $H^{1,1}(M,\mathbb{Z})$ are parabolic, hence virtually free abelian. ■

DEFINITION: The group $\ker\Psi$ is called **the group of symplectic automorphisms** of a K3 surface.

Automorphisms of projective K3 surfaces with small Picard rank

CLAIM: Let M be a projective K3 surface with Picard rank 1. **Then** Aut(M) **is finite.**

Proof: Indeed, for a projective K3, the map $\operatorname{Aut}(M) \to SO(H^{1,1}(M,\mathbb{Z}))$ has finite kernel (Proposition 1). ■

DEFINITION: Let (Λ, q) be a quadratic lattice, that is, $\mathbb{Z}^n$ equipped with an integer-valued quadratic form. We say that (Λ, q) **represents** $n \in \mathbb{Z}$ if there exists $x \in \Lambda$ such that $q(x,x) = n$.

CLAIM: Let M be a projective K3 surface with Picard rank 2. **Then** Aut(M) **is finite if its Neron-Severi lattice represents zero or** -2**,** and **contains an infinite cyclic subgroup of finite index otherwise.**

Proof: Since M is projective, Aut(M) is finite if and only if its group $\operatorname{Aut}_\Omega(M) \subset SO(H^{1,1}(M,\mathbb{Z}))$ of symplectic automorphisms is finite (Proposition 1). The group of isometries of a quadratic lattice (Λ, q) of signature (1,1) is finite if (Λ, q) represents zero. It contains an infinite cyclic subgroup of finite index otherwise, by Pell's classification of solutions of an equation $x^2 - ay^2 = 1$. ■

Reflections in the Neron-Severi lattice

DEFINITION: Let $v \in H^{1,1}(M,\mathbb{Z})$ be a (-2)-class. Consider the map $x \mapsto x - 2\frac{(x,v)}{(v,v)}v$, where $(\cdot,\cdot)$ is the intersection form. This map is called **the divisorial reflection**.

CLAIM: The group generated by divisorial reflections **acts transitively on the set of connected components of $\operatorname{Pos}(M)\backslash \bigcup_{s\in\mathfrak{S}} s^\perp$.**

Proof: Each wall separating the connected components is the fixed point set of a divisorial reflection, which exchanges two adjacent connected components. ■

REMARK: We denote the subgroup of $SO(H^2(M,\mathbb{Z}))$ generated by divisorial reflections by $\operatorname{Ref}(M)$. **Clearly, it is a normal subgroup in the group of all Hodge isometries of $H^2(M,\mathbb{Z})$.**

COROLLARY: The group $\operatorname{Aut}_\Omega(M)$ of symplectic automorphisms of a K3 surface **is determined, up to an isomorphism, by its Neron-Severi quadratic lattice.**

Proof: It is identified with the subgroup $\operatorname{St}(\operatorname{Kah}(M))$ of $SO(H^{1,1}(M,\mathbb{Z}))$ preserving the Kähler cone. However, $\operatorname{Ref}(M)$ acts on $SO(H^2(M,\mathbb{Z}))$ by Hodge isometries, transitively on all connected components of $\operatorname{Pos}(M)\backslash \bigcup_{s\in\mathfrak{S}} s^\perp$. Therefore, the action of $\operatorname{Ref}(M)$ is transitive on all groups of form $\operatorname{St}(A)$, where A is a connected component in $\operatorname{Pos}(M)\backslash \bigcup_{s\in\mathfrak{S}} s^\perp$, and $\operatorname{St}(A)$ its stabilizer. ■

The hyperbolic space and its isometries

REMARK: The group $O(m,n), m,n > 0$ has 4 connected components. We denote the connected component of 1 by $SO^+(m,n)$. We call a vector v **positive** if its square is positive.

DEFINITION: Let V be a vector space with quadratic form q of signature $(1,n)$, $\operatorname{Pos}(V) = \{x \in V \mid q(x,x) > 0\}$ its **positive cone**, and $\mathbb{P}^+V$ projectivization of $\operatorname{Pos}(V)$. Denote by g any $SO(V)$-invariant Riemannian structure on $\mathbb{P}^+V$. Then $(\mathbb{P}^+V, g)$ is called **hyperbolic space**, and the group $SO^+(V)$ **the group of oriented hyperbolic isometries**.

EXAMPLE: For any K3 surface, $\mathbb{P}\operatorname{Pos}(M)$ **is a hyperbolic space,** and $\operatorname{Aut}(M)$ **acts on this space by isometries.**

Sterk's theorem

THEOREM: (Sterk, 1985)
Let M be a projective K3 surface. **Then** $\operatorname{Aut}(M)$ **acts with finitely many orbits on on the set of the walls of the Kähler cone.**

Proof: For any quadratic lattice (Λ, q), the group $SO(\Lambda)$ acts on the set of vectors of fixed length with finitely many orbits (Kneser). Therefore, $SO(H^{1,1}(M, \mathbb{Z}))$ acts with finitely many orbits on the set of all (-2)-vectors. Applying this argument to the isometries preserving the Kähler cone, **we obtain that** $\operatorname{Aut}(M)$ **acts with finitely many orbits on the set of the walls of the Kähler cone.** ■

Morrison-Kawamata conjecture

REMARK: When M is a Calabi-Yau manifold, Sterk's theorem is known as **the Morrison-Kawamata conjecture**, formulated by Morrison in 1993, and proven by Kawamata for elliptic Calabi-Yau manifolds in 1997. It was proven by B. Totaro for log-Calabi-Yau surfaces (2010), and by Amerik-V. for hyperkähler manifolds (2015).

COROLLARY: Let $\operatorname{Amp}(M) := \operatorname{Kah}(M) \cap H^{1,1}(M,\mathbb{Q})$ denote **the ample cone** of a projective K3 surface, and $\operatorname{Amp}_{\mathbb{R}}(M) := \operatorname{Kah}(M) \cap H^{1,1}(M,\mathbb{Q}) \otimes_{\mathbb{Q}} \mathbb{R}$ denote the corresponding cone in $H^{1,1}(M,\mathbb{Q}) \otimes_{\mathbb{Q}} \mathbb{R}$. Consider its projectivization $\mathbb{P}\operatorname{Amp}_{\mathbb{R}}(M)$ as a subset in the hyperbolic space $\mathbb{P}\operatorname{Pos}(M)$. **Then $\mathbb{P}\operatorname{Amp}_{\mathbb{R}}(M)$ has finite volume if and only if $\operatorname{Aut}(M)$ is finite.**

Proof: By construction, $\mathbb{P}\operatorname{Amp}_{\mathbb{R}}(M)$ is a hyperbolic polyhedron (with some vertices on absolute). If $\operatorname{Aut}(M)$ is finite, this polyhedron has finite number of faces, hence its volume is finite. Conversely, if $\operatorname{Aut}(M)$ is infinite, $\mathbb{P}\operatorname{Amp}_{\mathbb{R}}(M)$ contains infinite number of copies of its fundamental domain, hence its volume is infinite. ■

K3 surfaces with finite automorphism groups

THEOREM: (Vinberg) Let M be a projective K3 surface of Picard rank $\geqslant 3$. **Then** $\operatorname{Aut}(M)$ **is finite if and only if the reflection group** $\operatorname{Ref}(M)$ **has finite index in** $SO(H^{1,1}(M,\mathbb{Z}))$.

Proof. Step 1: If $\operatorname{Aut}(M)$ is finite, the ample cone has finite volume, hence the action of $\operatorname{Ref}(M)$ on the hyperbolic space $\mathbb{P}\operatorname{Amp}_{\mathbb{R}}(M)$ has finite volume. This implies that $\operatorname{Ref}(M)$ is a lattice subgroup in $SO(H^{1,1}(M,\mathbb{R}))$, hence has finite index in $SO(H^{1,1}(M,\mathbb{Z}))$.

Step 2: Conversely, assume that $\operatorname{Ref}(M)$ has finite index in $SO(H^{1,1}(M,\mathbb{Z}))$. Then its Coheter polyhedron is finite (Vinberg). However, $\mathbb{P}\operatorname{Amp}_{\mathbb{R}}(M)$ is its Coxeter polyhedron. ■

Reflective lattices

DEFINITION: We call a quadratic lattice with $\operatorname{Ref}(\Lambda)$ of finite index in $SO(\Lambda)$ **reflective**.

COROLLARY: Let $\mathfrak{R}_1$ be the set of of Hodge lattices $\Lambda\subset H^2(M,\mathbb{Z})$ such that the corresponding K3 surface is projective, has Picard rank $\geqslant 3$, and has finite automorphism group, and $\mathfrak{R}$ the set of such lattices up to $\operatorname{Diff}(M)$-action. **Then $\mathfrak{R}$ is finite.**

Proof: By Vinberg's theorem, the set of isomorphism classes of primitive reflective lattices is finite. Therefore, the set of isomorphism classes of the lattices $\Lambda = H^{1,1}(M,\mathbb{Z})$ with $|\operatorname{Aut}(M)|<\infty$ is also finite. However, the group $SO(H^2(M,\mathbb{Z}))$ acts on all isometric embeddings $\Lambda\hookrightarrow H^2(M,\mathbb{Z})$ with finitely many orbits (Kneser). ■

Noether-Lefschetz components for reflective lattices

COROLLARY: Let H be the moduli space of polarized K3 surfaces. **Then there is a finite set of quasiprojective subvarieties $Z_1, ..., Z_n \subset H$ such that for any K3 surface M outside of Z_i with Picard rank $\geqslant 3$, the group $\operatorname{Aut}(M)$ is infinite.**

Proof. Step 1: Fix a polarization on M, that is, an integer vector $p \in H^2(M,\mathbb{Z})$ such that its square is positive. By Torelli theorem, $H = \frac{\mathbb{P}\mathrm{er}}{SO(H^2(M,\mathbb{Z}),p)}$, where $\mathbb{P}\mathrm{er}$ is the space of all Hodge structures on $H^2(M)$ such that p has type (1,1), and $SO(H^2(M,\mathbb{Z}),p)$ the group of isometries preserving p. For any sublattice $\Lambda \subset H^2(M,\mathbb{Z})$ containing p, the corresponding **Noether-Lefschetz component** $H_\Lambda \subset H$ is the image of all $I \in \mathbb{P}\mathrm{er}$ such that $H^{1,1}(M,I) \supset \Lambda$. By Baily-Borel theorem, H_Λ is a quasiprojective subvariety in H.

Step 2: Let Z_i be the set of Noether-Lefschetz components associated with reflective lattices $\Lambda \subset H^2(M,\mathbb{Z})$. Clearly, any M which has Picard rank $\geqslant 3$ and not in Z_i has infinite automorphism group. ■

Families of K3 surfaces with finite automorphism groups

THEOREM: (K. Oguiso, 2003)
Let M_t be a non-trivial quasiprojective family of K3 surfaces over a curve C, with general member of Picard rank $\geqslant 3$. **Then $|\operatorname{Aut}(M_t)| = \infty$ for a dense set of $t \in C$.**

Proof: Remove from C all proper subsets which belong to the Noether-Lefschetz components Z_i. If $C \not\subset Z_j$, we are done. Otherwise, there is a dense subset of points in C where the Picard rank jumps, and these points are not in Z_i. ■

REMARK: This theorem is also valid for families with general member of Picard rank $\geqslant 1$, but the proof for lower Picard values is more elementary.

REMARK: Oguiso's proof does not use Vinberg's theorem, he uses explicit lattice-theoretic arguments.

Hyperkähler manifolds

DEFINITION: A holomorphically symplectic manifold is a complex manifold equipped with non-degenerate, holomorphic $(2,0)$-form.

DEFINITION: For the rest of this talk, **a hyperkähler manifold is a compact, Kähler, holomorphically symplectic manifold.**

DEFINITION: A hyperkähler manifold M is called **of maximal holonomy**, or **IHS**, if $\pi_1(M)=0$, $H^{2,0}(M)=\mathbb{C}$.

Bogomolov's decomposition: Any hyperkähler manifold admits a finite covering which is a product of a torus and several simple hyperkähler manifolds.

Further on, all hyperkähler manifolds are assumed to be of maximal holonomy.

Hyperkähler manifold without birational automorphisms

THEOREM: (Denisi, Onorati, Rizzo, Viktorova)
Let M_t be a non-trivial quasiprojective family of hyperkähler manifolds of known type over a curve C. **Then $|\operatorname{Bir}(M_t)| = \infty$ for a dense set of $t \in C$,** where $\operatorname{Bir}(M_t)$ denotes the group of birational automorphisms.

REMARK: Their proof relies on known properties of the cohomology lattices of hyperkähler manifolds.

THEOREM: (Amerik, Soldatenkov, V.)
This theorem holds for any family of hyperkähler manifolds.

REMARK: Our proof is based on Vinberg's theorem, in fact, it is pretty much identical to the proof for K3 surfaces that I gave.

The Bogomolov-Beauville-Fujiki form

THEOREM: (Fujiki). Let $\eta \in H^2(M)$, and $\dim M = 2n$, where M is hyperkähler. **Then $\int_M \eta^{2n} = cq(\eta,\eta)^n$,** for some primitive integer quadratic form q on $H^2(M,\mathbb{Z})$, and $c > 0$ a rational number.

Definition: This form is called **Bogomolov-Beauville-Fujiki form**. **It is defined by the Fujiki's relation uniquely, up to a sign.** The sign is determined from the following formula (Bogomolov, Beauville)

$$\begin{aligned}\lambda q(\eta,\eta) = 2\int_X \eta\wedge\eta\wedge\Omega^{n-1}\wedge\overline{\Omega}^{n-1} - \\ -\frac{n-1}{n}\left(\int_X \eta\wedge\Omega^{n-1}\wedge\overline{\Omega}^{n}\right)\left(\int_X \eta\wedge\Omega^{n}\wedge\overline{\Omega}^{n-1}\right)\end{aligned}$$

where Ω is the holomorphic symplectic form, and $\lambda > 0$.

Remark: **q has signature $(3, b_2 - 3)$.** It is negative definite on primitive forms, and positive definite on $\langle\Omega,\overline{\Omega},\omega\rangle$, where ω is a Kähler form.

Unlike $H^2(K3,\mathbb{Z})$, **the BBF form is usually not unimodular**.

Monodromy group of a hyperkähler manifold

DEFINITION: Let M be a hyperkähler manifold, and $\operatorname{Mon}(M)$ the group of automorphisms of $H^2(M)$ generated by monodromy transform for all Gauss-Manin local systems. Then $\operatorname{Mon}(M)$ is called **the monodromy group of** M.

Theorem 1:
The group $\operatorname{Mon}(M) \subset O(H^2(M,\mathbb{Z})$ **has finite index in** $O(H^2(M,\mathbb{Z}))$**.**

THEOREM: Let M be a hyperkähler manifold, $\operatorname{Mon}(M)$ the group of automorphisms of $H^2(M)$ generated by monodromy transform for all Gauss-Manin local systems, and $\operatorname{Mon}_I(M)$ the Hodge monodromy group, that is, a subgroup of $\operatorname{Mon}(M)$ preserving the Hodge decomposition. **Then** $\operatorname{Aut}(M)$ **surjects to the subgroup of** $\operatorname{Mon}_I(M)$ **preserving the Kähler cone** $\operatorname{Kah}(M)$**,** and the kernel of this map is finite.

Proof: Follows from the global Torelli theorem.

MBM classes

DEFINITION: Negative class on a hyperkähler manifold is $\eta \in H_2(M, \mathbb{R}) = H^2(M, \mathbb{R})$ satisfying $q(\eta, \eta) < 0$. It is **effective** if it is represented by a curve.

THEOREM: Let $z \in H_2(M, \mathbb{Z})$ be negative, and I, I' complex structures in the same deformation class, such that z is of type (1,1) with respect to I and I' and $\mathrm{Pic}(M) = \langle z \rangle$. Then $\pm z$ **is effective in** (M, I) $\Leftrightarrow$ **iff it is effective in** (M, I').

REMARK: From now on, we identify $H^2(M)$ and $H_2(M)$ using the BBF form. Under this identification, **integer classes in** $H_2(M)$ **correspond to rational classes in** $H^2(M)$ (the form q is not unimodular).

DEFINITION: A negative class $z \in H^2(M, \mathbb{Z})$ on a hyperkähler manifold is called **an MBM class** if there exist a deformation of M with $\mathrm{Pic}(M) = \langle z \rangle$ such that λz is represented by a curve, for some $\lambda \neq 0$.

MBM classes and the shape of the Kähler cone

THEOREM: Let (M,I) be a hyperkähler manifold, and $S \subset H_{1,1}(M,I)$ the set of all MBM classes in $H_{1,1}(M,I)$. Consider the corresponding set of hyperplanes $S^\perp := \{W = z^\perp \mid z \in S\}$ in $H^{1,1}(M,I)$. **Then the Kähler cone of (M,I) is a connected component of** Pos$(M,I)\backslash \cup S^\perp$, where Pos$(M,I)$ is a positive cone of (M,I). Moreover, for any connected component K of Pos$(M,I)\backslash \cup S^\perp$, there exists $\gamma \in O(H^2(M))$ in a monodromy group of M, and a hyperkähler manifold (M,I') birationally equivalent to (M,I), such that $\gamma(K)$ is a Kähler cone of (M,I').

REMARK: This implies that **MBM classes correspond to faces of the Kähler cone.**

DEFINITION: Kähler chamber is a connected component of Pos$(M,I)\backslash \cup S^\perp$.

CLAIM: The Hodge monodromy group maps Kähler chambers to Kähler chambers.

Birational Kähler cone

REMARK: Define **pseudo-isomorphism** $M \longrightarrow M'$ as a birational map which is an isomorphism outside of codimension $\geqslant 2$ subsets of M, M'.
REMARK: For any pseudo-isomorphic manifolds M, M', one has $H^2(M) = H^2(M')$.

DEFINITION: Movable Kähler cone, also known as **birational Kähler cone**, is a closure of the union of Kah(M') for all M' pseudo-isomorphic to M.

DEFINITION: A cohomology class $\nu \in H^{1,1}(M)$ is called **pseudoeffective** if it can be represented by a positive, closed current.

THEOREM: (Boucksom)
On any hyperkähler manifold M, **birational Kähler cone is dual (with respect to the BBF pairing) to the pseudoeffective cone.**

COROLLARY: Let $\eta \in$ Pos(M) be an element of a positive cone on a hyperkähler manifold. **Then η belongs to the birational Kähler cone if and only if $q(\eta, E) \geqslant 0$ for any exceptional divisor E.**

REMARK: In other words, **the faces of birational Kähler cone are dual to the classes of exceptional divisors.**

Divisorial MBM classes

THEOREM: (Amerik, V.)
Let $\eta \in H^{1,1}(M,\mathbb{Z})$ be an MBM class which lies on the boundary of the Kähler cone. **Then there exists a holomorphic bimeromorphic map $\varphi: M \to M'$ such that all its fibers are the unions of rational curves C such that $[C]$ is proportional to η.**

DEFINITION: The exceptional set of φ is called **the MBM center** associated with the MBM class η.

THEOREM: (Amerik, V.)
Let M_t be a family of hyperkähler manifolds such that $\eta \in H^{1,1}(M_t,\mathbb{Z})$ is an MBM class on the boundary of the Kähler cone for all t. Denote by E_t the corresponding MBM center. **Then all E_t are homeomorphic**.

DEFINITION: An MBM class is called **divisorial** if its MBM center is a divisor.

THEOREM: (Markman)
Let $v \in H^{1,1}(M,\mathbb{Z})$ be a divisorial MBM class. **Then the reflection map $x \mapsto x - 2\frac{(x,v)}{(v,v)}v$ belongs to the monodromy group**.

REMARK: If we replace the (-2)-classes by divisorial MBM classes, **the arguments that we used to prove Oguiso's theorem for K3 become valid for an arbitary hyperkähler manifold.**