Algebraic geometry: test assignment 1

Rules: Please solve this in class, 10:00-12:00, Tuesday 29.09.2015, and bring the results to my pigeonhole in the mail room at 9-th floor.

Exercise 1.1. Prove that the group of automorphisms of the field $\mathbb{R}$ (real numbers) over $\mathbb{Q}$ (rationals) is trivial.

Exercise 1.2. Let F_1, F_2 be finite fields with 8 and 16 elements. Find an embedding of F_1 to F_2, or prove that it does not exist.

Exercise 1.3. An element a of the field $\mathbb{Z}/p\mathbb{Z}$ is called **quadratic residue** if $a = x^2$ has a solution modulo p. Find the product of all non-zero quadratic residues in $\mathbb{Z}/p\mathbb{Z}$ for all p.

Definition 1.1. Let I be a set of functions on a space M. We denote by $V(I)$ the set of all points $x \in M$ where all $f \in I$ vanish ("the set of common zeros").

Exercise 1.4 (*). Let I be an ideal in the ring $C^0(M)$ of continuous, real-valued functions on a compact topological space M. Prove that $V(I)$ is non-empty.

Exercise 1.5. Show that there exists a non-trivial ideal I in the ring $C(\mathbb{Z})$ of functions on a discrete set $\mathbb{Z}$ with $V(I)$ empty.