

MATH-F-303, test assignment 1 (group theory)

Rules: Please solve this in class and give the solutions to Roberto. For the definitions, you may use the home assignment for 21.09.2015, distributed together with this test exercises. Problems with (*) are extra hard, and you get extra credit for solutions.

Exercise 1.1. Prove that all groups of order 4 are commutative.

Exercise 1.2. Let G be a group with all elements of order 2. Prove that it is commutative.

Exercise 1.3. Find a non-commutative group of order 8.

Exercise 1.4. Let G be a group of even order. Prove that G has an element of order 2.

Exercise 1.5. Let G be an infinite group. Prove that G contains infinitely many distinct subgroups.

Exercise 1.6 (*). Find two non-isomorphic non-commutative groups of order 8.

Exercise 1.7. Find a non-commutative group G with center Z such that G/Z is commutative.