

MATH-F-303, test assignment 10 (orthogonal group)

Rules: This is a class assignment for discussion.

Exercise 10.1. Consider the group $O(1, 1)$ of automorphisms of \mathbb{R}^2 preserving the metric of signature $(1,1)$. Prove that $O(1, 1)$ has at least 4 connected components.

Exercise 10.2. Prove that the connected component of $O(1, 1)$ is isomorphic to \mathbb{R} .

Exercise 10.3. Prove that the group $SO(n)$ is connected.

Exercise 10.4. Let $\gamma \in O(n)$ be any automorphism of a vector space preserving a scalar product. Prove that $\det \gamma = 1$.