MATH-F-303, assignment 11 (quadratic forms)

Rules: This is a class assignment for discussion.

Rules: Please solve this in class and give the solutions to Roberto. Problems with (*) are extra hard, and you get extra credit for solutions.

Definition 11.1. Let g be a bilinear symmetric form on a vector space V. The map $h(v) := g(v, v)$ is called **quadratic form**. Quadratic form is **diagonal** if $h(v) = \sum_{i=1}^n a_i \lambda_i(v)^2$, where λ_i is a basis in V^*. **Signature** of h (and of g) is the number of positive and negative coefficients a_i.

Exercise 11.1. Find signature of the quadratic form $h(x, y, z) = x^2 + y^2 + z^2 + 500yz$ (we consider it as a quadratic form on $\mathbb{R}^3$).

Exercise 11.2. Find signature of the quadratic form $h(x, y, z) = x^2 + y^2 + z^2 + 500xz + 500yz$.

Exercise 11.3. Find signature of the quadratic form $h(x, y, z) = x^2 + y^2 + z^2 + 500xz + 500yz + 500yz$.

Exercise 11.4. Let g be a quadratic form of signature (1,1) on $V = \mathbb{R}^2$, and $P : V \longrightarrow V$ a map preserving g (that is, satisfying $g(v) = g(P(v))$ for all $v \in V$). Assume that P belongs to connected component of $SO(1, 1)$. Prove that all eigenvalues of P are real.

Exercise 11.5 (*). Let g be a quadratic form of signature (1,2) on $V = \mathbb{R}^2$, and $P : V \longrightarrow V$ a map preserving g. Prove that P has one eigenvalue which is equal to ± 1.