MATH-F-303, test assignment 2 (rings and fields)

Rules: Please solve this in class and give the solutions to Roberto. For the definitions, you may use the home assignment for 28.09.2015, distributed together with this test exercises. Problems with (*) are extra hard, and you get extra credit for solutions.

Exercise 2.1. Let R be a ring with p elements, where p is prime. Prove that R is isomorphic to $\mathbb{Z}/p\mathbb{Z}$ (ring of remainders modulo p).

Exercise 2.2. Prove that any field contains the field of rationals $\mathbb{Q}$ or the field of reminders $\mathbb{Z}/p\mathbb{Z}$.

Exercise 2.3 (*). Let σ be an automorphism of the field of reals $\mathbb{R}$. Prove that σ is identity.

Exercise 2.4. Find 3 non-isomorphic rings with 8 elements.

Exercise 2.5. Let R be a ring with 15 elements. Prove that R is $\mathbb{Z}/15\mathbb{Z}$

Exercise 2.6 (*). Prove that $(p-1)! = p-1 \mod p$ (same remainder modulo p).

Exercise 2.7. Let A be a finite ring without zero divisors (that is, without non-zero elements x, y such that $xy = 0$). Prove that A is a field.