

## MATH-F-303, test assignment 3 (more fields)

**Rules:** This is a class assignment for discussion.

**Exercise 3.1.** Let  $F$  be a finite field. Prove that the number of elements in  $F$  is  $p^k$ , where  $p$  is characteristic of  $F$ .

**Exercise 3.2.** Find a polynomial which is irreducible over the field  $\mathbb{Q}[\sqrt{-1}]$ .

**Exercise 3.3.** Prove that the group  $(\mathbb{Z}/p\mathbb{Z})^*$  of non-zero remainders modulo prime  $p$  is cyclic.

**Exercise 3.4 (\*).** Let  $p$  be prime,  $p \neq 2$ . **Quadratic residue** is an element of  $\mathbb{Z}/p\mathbb{Z}$  which is a square. Prove that  $-1$  is a quadratic residue mod  $p$  if and only if  $p \equiv 1 \pmod{4}$ .

**Exercise 3.5.** Find the sum of all quadratic residues in  $\mathbb{Z}/17\mathbb{Z}$ .

**Exercise 3.6.** Let  $l$  be a positive integer, not divisible by 2 and 5. Prove that there exists  $k$  such that  $10^k - 1$  is divisible by  $l$ .