

MATH-F-303, simple test assignment 5 (tensor product)

Rules: Please solve this in class and give the solutions to Roberto. Please sign the papers. Your results go towards your final grade.

Exercise 5.1. Let $\dim V = 3$, $\dim W = 5$. Find dimension of $\text{Hom}(V, W) \otimes W$.

Exercise 5.2. Let $\dim V = 3$, $\dim W = 5$. Find dimension of $\text{Hom}(V, \text{Hom}(W, W))$.

Definition 5.1. Symmetric group Σ_n acts on $V^{\otimes n} := \underbrace{V \otimes V \otimes \dots \otimes V}_{n \text{ times}}$ exchanging the components. Denote by $\text{Sym}^n V$ the Σ_n -invariant part of $V^{\otimes n}$.

Exercise 5.3. Let V be a 2-dimensional space. Find dimension of $\text{Sym}^n V$.

Exercise 5.4. The space of Cartan tensors $C(V)$ is the space of tensors $\psi \in V \otimes V \otimes V$ which are symmetric under exchange first two components and antisymmetric under exchange of the last two. Find dimension of $C(V)$ when $\dim V = 1$

Exercise 5.5. Let $\dim V = 2$, $\dim W = 10$. Find the dimension of the space of bilinear symmetric forms $V \otimes V \rightarrow W$ (with values in W).

Exercise 5.6. Let $\dim V = 2$, $\dim W = 10$. Find the dimension of the space of bilinear anti-symmetric forms $V \otimes V \rightarrow W$ (with values in W).