

MATH-F-303, class assignment 6 (algebras with generators and relations)

Rules: This is a class assignment for discussion.

Exercise 6.1. Let A be an algebra over \mathbb{C} generated by x, y with relations $x^3 = y^3 = 0$, $xy = yx$. Find its dimension.

Exercise 6.2. Let A be an algebra over \mathbb{C} generated by x, y with relations $x^2 = 0$, $y^2 = 0$, $xy - yx = 1$. Find its dimension.

Exercise 6.3. Let A be an algebra over a field k generated by x . Prove that either A is finite-dimensional over k , or A is isomorphic to the polynomial algebra $k[t]$.

Exercise 6.4. Let A be an algebra over \mathbb{R} generated by x, y with relations $x^2 = -1$, $y^2 = -1$, $xy = -yx$. Prove that it is isomorphic to quaternions.

Exercise 6.5. Let A be an algebra over \mathbb{R} generated by x, y with relations $x^2 = 1$, $y^2 = 1$, $xy = -yx$. Prove that it is isomorphic to the matrix algebra $\text{Mat}(2, \mathbb{R})$.