MATH-F-303, a simple test assignment 7 (group representations)

Rules: Please solve this in class and give the solutions to Roberto. Please sign the papers. Your results contribute towards your final grade.

Definition 7.1. Let V be a vector space. We denote by $GL(V)$ the group of all invertible linear maps from V to itself. Let $\rho: \ G \longrightarrow GL(V)$ a group homomorphism. Then ρ is called **representation of** G, and V **the space of representation** ρ. We denote the action $v \mapsto \rho(g)(v)$ by $v \mapsto g(v)$. In this situation, V^G denotes the space of G-invariant vectors.

Exercise 7.1. Let Σ_n act on $V = \mathbb{R}^n$ by permitations of coordinates. Find the dimension of the space of invariants V^{Σ_n}.

Exercise 7.2. Let Σ_2 act on $V = \mathbb{R}^2$ by permitations of coordinates. Extend this actopn to $V \otimes V$ by $g(v \otimes v') = g(v) \otimes g(v')$ Find the dimension of the space of invariants $(V \otimes V)^{\Sigma_2}$.

Exercise 7.3. Consider the action of the cyclic group $\mathbb{Z}/n\mathbb{Z}$ on $V = R^n$ given by cyclic permutations of coordinates. Find the dimension of the space of invariants $V^{\mathbb{Z}/n\mathbb{Z}}$.

Exercise 7.4. Let $G = (\mathbb{Z}/2)^n$ be the group acting on $V = \mathbb{R}^n$ by flipping the sign of coordinates (with the generator $(0, 0, ..., 1, ..., 0)$, 1 at i-th place mapping $(x_1, ..., x_i, ..., x_n)$ to $(x_1, ..., -x_i, ..., x_n)$. Find the dimension of the space of invariants $V^{\mathbb{Z}/n\mathbb{Z}}$.

Exercise 7.5. Let $G = (\mathbb{Z}/2)^2$ be the group acting on $V = \mathbb{R}^2$ by flipping the sign of coordinates. Find the dimension of the space of invariants $(V \otimes V)^G$.

Exercise 7.6 (*). Let $G := \{\pm 1\}$ act on $V = \mathbb{C}^2$ by mapping (x, y) to $(-x, -y)$. Find the dimension of $(\mathsf{Sym}^i V)^G$ for all i.