Riemann surfaces: test assignment 1

Rules: Please solve this in class, 10:00-12:00, Wednesday 30.09.2015, and bring the results to my pigeonhole in the mail room at 9-th floor.

Exercise 1.1. Let f be a complex analytic function on $\mathbb{C}$ such that there exists $n > 0$ such that $\frac{|f(z)|}{|z|^n}$ is bounded for all z with $|z| > 1$. Prove that f is a polynomial.

Exercise 1.2. Let $f \in \mathbb{C}[t]$ be a polynomial, considered as a function on $\mathbb{C}$. Prove that $\frac{1}{2\pi}\int_{\partial\Delta} f d\mu = f(0)$, where $\partial\Delta$ is a unit circle in $\mathbb{C}$ and $d\mu$ the standard measure.

Exercise 1.3. Let f be a complex-valued continuous function function on an open subset $U \subset \mathbb{R}^n$ such that $\int_U d\mu = 1$, where $d\mu$ is the standard measure. Prove that $\left|\int_U f d\mu\right| \leqslant \sup_U |f|$, and equality is realized only when f is constant.

Definition 1.1. An **almost complex structure** on a manifold M is a map $I: TM \longrightarrow TM$ such that $I^2 = -\mathsf{Id}$.

Exercise 1.4. Prove that any compact 2-dimensional real manifold admits an almost complex structure, or find a counterexample

Exercise 1.5. Let G be a Lie group (that is, a smooth manifold equipped with a group structure, such that the group operations are given by smooth maps). Prove that $G \times G$ admits an almost complex structure.