Riemann surfaces: test assignment 2

Rules: Please bring the results to my pigeonhole in the mail room at 9-th floor. Problems with (*) are extra hard.

Exercise 2.1. Let f be a holomorphic function on disk $\Delta \subset \mathbb{C}$ which is continuoulsly extended to the boundary $\partial\Delta$. Suppose that $f\big|_{\partial\Delta}$ takes values in $\mathbb{R}$. Prove that f is constant, or find a counterexample.

Exercise 2.2. Let ρ be a real 2-form on a vector space V equipped with a complex structure operator I. Assume that $\rho(x, Iy) = \rho(Ix, y)$. Prove that ρ is a real part of $(2,0)$-form.

Exercise 2.3 (*). Let f a real-valued smooth function on a simply connected Riemann surface which satisfies $dId(f) = 0$. Prove that f is a real part of a holomorphic function.

Exercise 2.4. Let $f: M \longrightarrow M$ be a conformal homeomorphism of a Hermitian Riemann surface. Assume that f preserves the Riemannian volume. Prove that f is an isometry.

Exercise 2.5. Let G be a finite group acting on a smooth manifold M, and Z the set of fixed points of G-action. Prove that Z is a smooth submanifold.