Riemann surfaces: homework assignment 2

Rules: Discussed in class on the next lecture

Exercise 2.1. Let f be a holomorphic function on disk $\Delta \subset \mathbb{C}$ which is continuoulsly extended to the boundary $\partial\Delta$. Suppose that $f\big|_{\partial\Delta}$ takes values in $\mathbb{R}$. Prove that f is constant, or find a counterexample.

Exercise 2.2. Let ρ be a real 2-form on a vector space V equipped with a complex structure operator I. Assume that $\rho(x, Iy) = \rho(Ix, y)$. Prove that ρ is a real part of $(2, 0)$-form.

Definition 2.1. **Riemannian surface** is a 2-dimensional Riemannian manifold. A map of Riemannian manifolds is **conformal** if it multiplies the metric by a function.

Exercise 2.3. Let $f: M \longrightarrow M$ be a conformal homeomorphism of a Hermitian Riemann surface. Assume that f preserves the Riemannian volume. Prove that f is an isometry.

Exercise 2.4. Let $M := G/H$, where $G = SO(2, n)$ (the Lie group of orthogonal automorphisms of a real space of signature $(2, n)$), and $H = SO(2) \times SO(n)$ embedded to G in the standard way. Prove that M admits a G-invariant almost complex structure.

Exercise 2.5. Let $M := G/H$, where $G = SO(2+n)$, and $H = SO(2) \times SO(n)$ embedded to G in the standard way. Prove that M admits a G-invariant almost complex structure.