Riemann surfaces: homework assignment 3

Rules: Discussed in class on the next lecture

Exercise 3.1. Let d be a metric on a disk Δ with the following properties: (a) d is $PSL(2,\mathbb{R})$-invariant and (b) d is continuous in the usual topology on $\Delta\times\Delta$. Prove that d is unique up to a constant multiplier.

Exercise 3.2. Let $\gamma = \frac{az+b}{cz+d}$ be an automorphism of Δ fixing a point $x \in \Delta$. Assume that a, b, c, d are integers. Prove that γ has finite order.

Exercise 3.3. Let $\gamma = \frac{az+b}{cz+d}$ be an automorphism of Δ without fixed points in Δ. Prove that γ has a fixed point on the boundary $\partial\Delta$.

Exercise 3.4. Let $\gamma = \frac{az+b}{cz+d}$ be an automorphism of Δ without fixed points in Δ. Find examples when γ has exactly one fixed point and exactly two fixed points on $\partial\Delta$. Prove that any automorphism fixing 3 points on the closure $\bar{\Delta}$ is an identity.